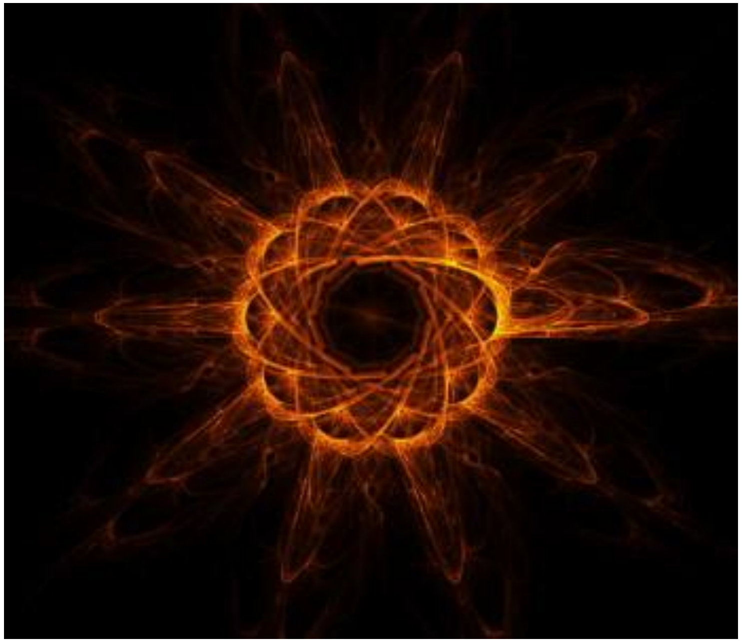
University of Cape Town
Department of Physics

PHY3022S
Nuclear and Particle Physics



Nuclear Physics Part 2
Nuclear reactions

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Energy released in a reaction

Consider the reaction $A + a \rightarrow B + b$ or $A(a,b)B$

In a nuclear reaction, binding energy is released, becoming kinetic energy of the decay products…

$$\textbf{\textit{Q}-value} = \{(M_A + M_a) - (M_B + M_b)\}c^2$$

By conservation of energy:
(if A is at rest in COM) $M_A c^2 + M_a c^2 + T_a = M_B c^2 + T_B + M_b c^2 + T_b$

Then $Q = T_B + T_b - T_a$

Q-values can be positive (exothermic reaction)
negative (endothermic reaction)
zero (elastic scattering)

Energy released in radioactive decay

In nuclear decay, binding energy is released, becoming kinetic energy of the decay products…

$$Q = \text{energy released in the decay}$$
$$= \left\{ \left(\sum \text{mass}\right)_{\text{before}} - \left(\sum \text{mass}\right)_{\text{after}} \right\} c^2$$

$Q > 0$ is a <u>condition</u> for the decay to occur.

For example, consider the α–decay: ${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$

The Q–value or disintegration energy is

$$Q = M_{\text{X}}c^2 - (M_{\text{Y}}c^2 + M_{\alpha}c^2)$$

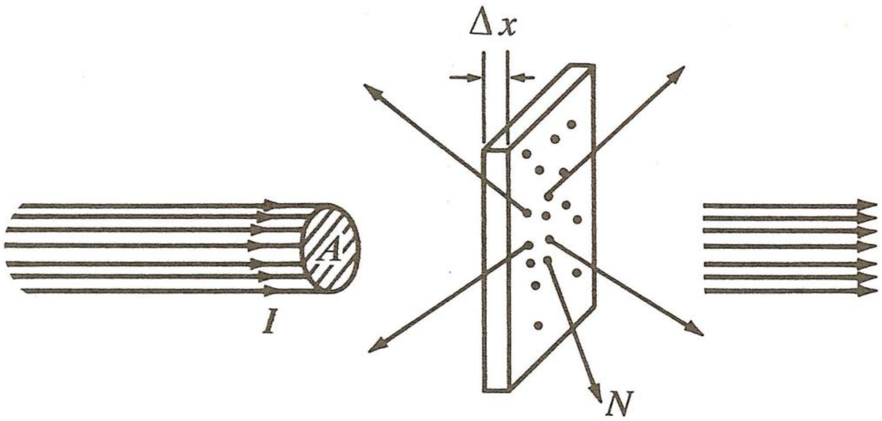
This can be shown to be the same as:

$$Q = B_{\text{X}} - B_{\text{Y}} - B_{\alpha}$$
$$= A\left(\frac{B_{\text{X}}}{A}\right) - (A-4)\left(\frac{B_{\text{Y}}}{A-4}\right) - 4\left(\frac{B_{\alpha}}{4}\right)$$

Nuclear reaction cross section

If we scatter a beam from a target, then what is the reaction rate?

Mono-energetic beam of
I particles per unit time,
cross section *A*,
hitting a thin slab of
thickness Δx, producing
N particles per unit time …



Probability of any one bombarding particle producing a hit = N/I

Introduce a "**reaction cross section**" σ
… which is a measure of the probability of a reaction taking place.

Then $$\frac{N}{I} = \frac{nA\Delta x\sigma}{A}$$

where *n* is the number of target nuclei per unit volume

Nuclear reaction cross section continued

$$\sigma = \frac{N}{(I/A)(nA\Delta x)}$$

Unit of cross section σ is the **barn** b, where 1 b = 10^{-24} cm^2

For a thin target, the yield of light products is then $N = n\sigma\Delta x I$

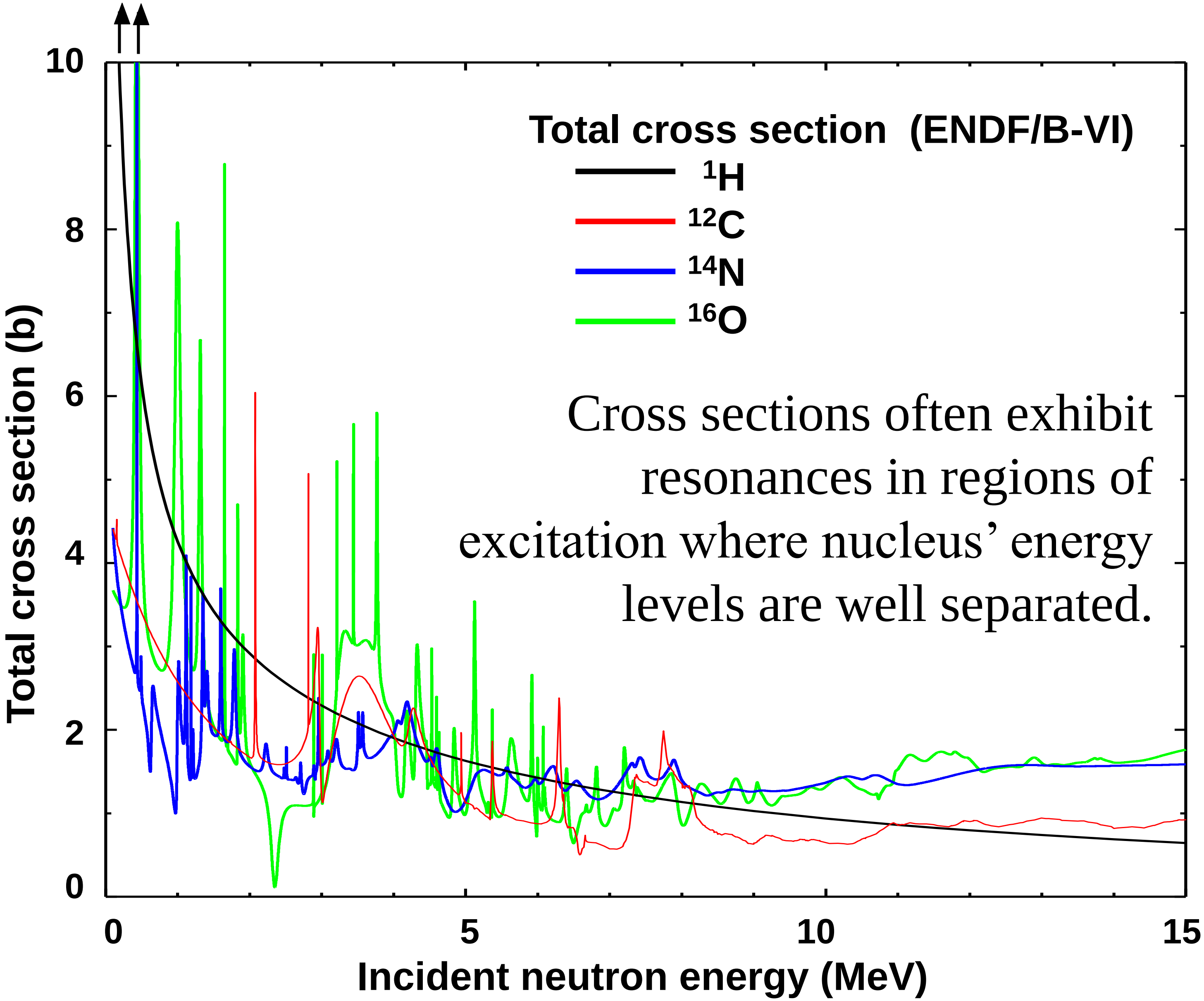
If the target is made of nuclei of atomic mass M_A, density ρ, then

$$N = I\sigma(\rho\Delta x)N_A/M_A$$

N_A: Avogadro's number

For a thick target of thickness t , $dN = -n\sigma dx I$

Then $I_t = I_0 e^{-n\sigma t}$



Cross sections often exhibit resonances in regions of excitation where nucleus' energy levels are well separated.

Nuclear reaction cross section continued

A given bombarding particle and target can often interact in a variety of ways.

Total cross section is then $$\sigma_{total} = \frac{N_1 + N_2 + N_3 + \ldots}{(I/A)(nA\Delta x)}$$

Partial cross section: $$\sigma_i = \frac{N_i}{(I/A)(nA\Delta x)}$$

Then $$\sigma_{total} = \sum_i \sigma_i$$

For a thick slab $$\frac{N_i}{I_0} = \frac{\sigma_i}{\sigma_{total}}\left(1 - e^{-n\sigma_{total}t}\right)$$

Differential cross section

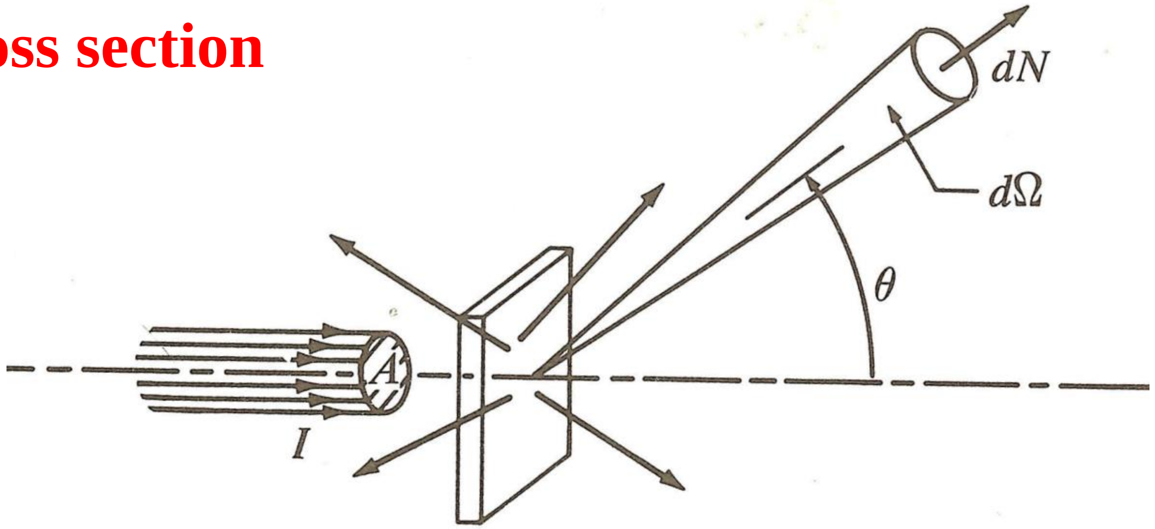
In many nuclear reactions, the light products are not produced isotropically, so introduce a differential cross section.

If $$\frac{N}{I} = \frac{nA\Delta x\sigma}{A}$$ then $$\frac{1}{I}\frac{dN}{d\Omega} = \frac{nA\Delta x}{A}\frac{d\sigma}{d\Omega}$$

Write **differential cross section**

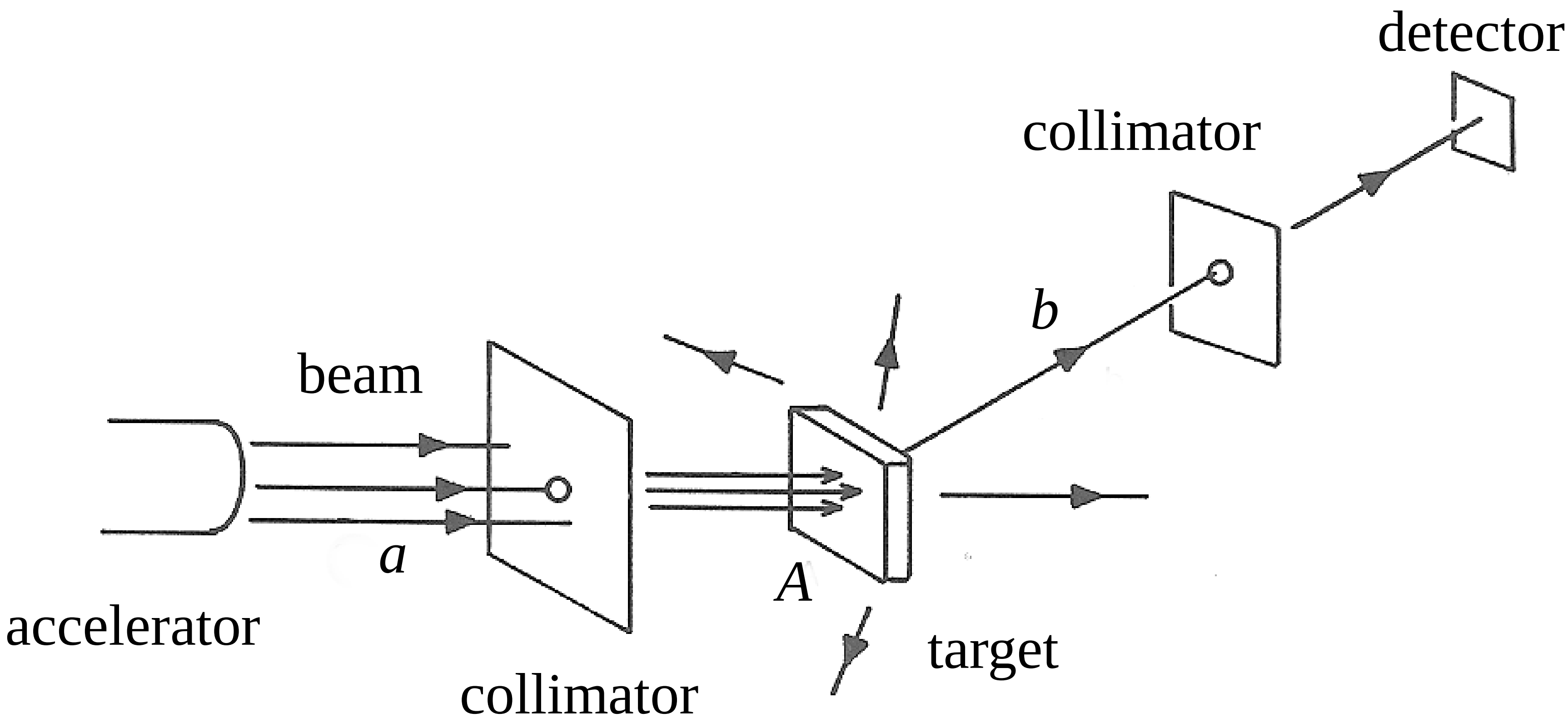
$$\frac{d\sigma}{d\Omega} = \frac{dN/d\Omega}{nA\Delta x(I/A)}$$

units: b sr^{-1}

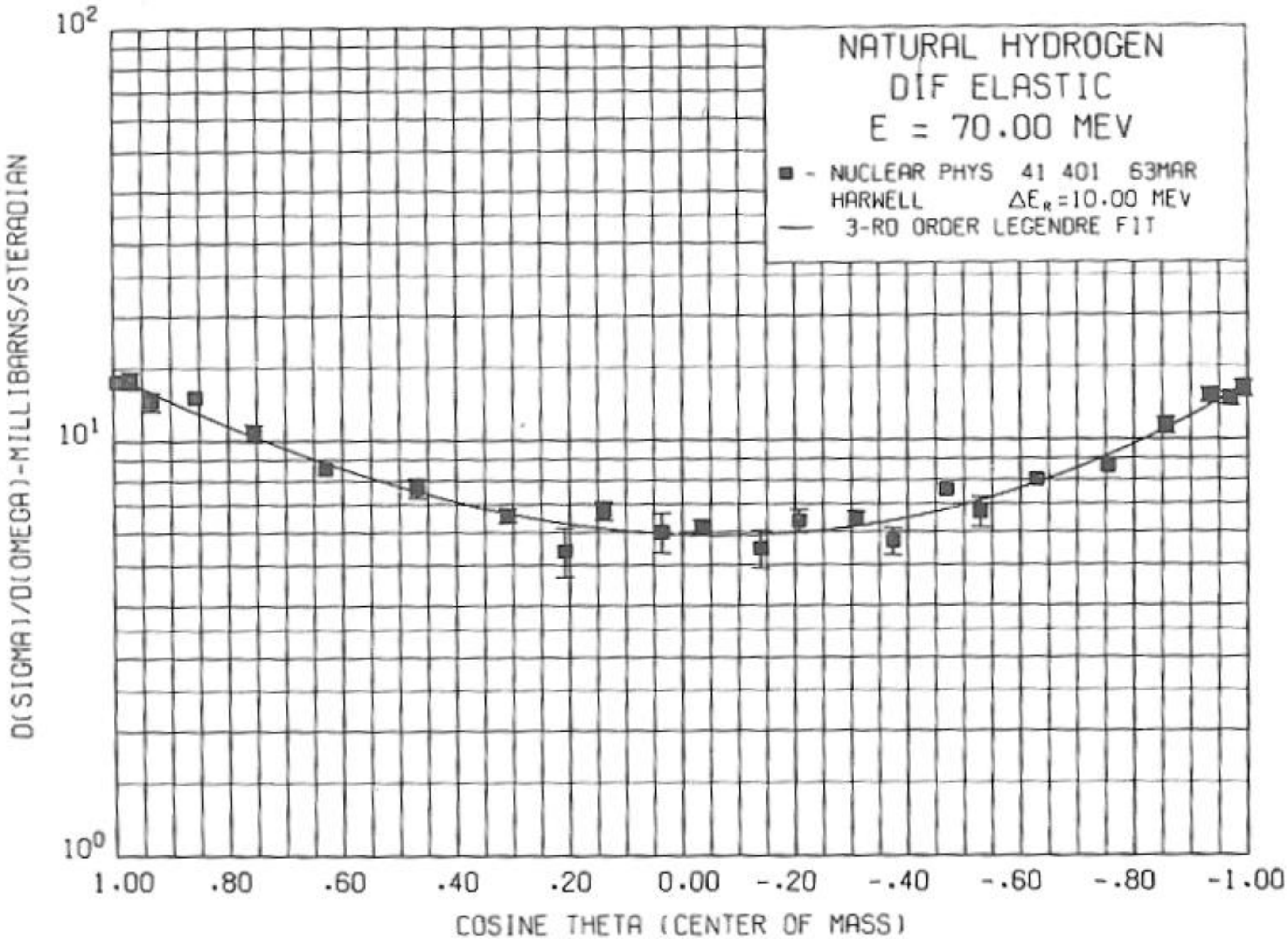


Then $$\sigma = \int_{\text{all space}} \frac{d\sigma}{d\Omega} d\Omega = \int_0^{2\pi} \int_{-1}^{+1} \frac{d\sigma}{d\Omega} d\cos\theta$$

An experimental approach to nuclear reactions



Differential cross section for H(n,n)H at E_n = 70 MeV



Nuclear reactions

A simple classification …

Elastic scattering: initial and final particles are identical

$$a + \mathrm{A} \rightarrow a + \mathrm{A}$$

Inelastic scattering: one or more of outgoing particles are in an excited state

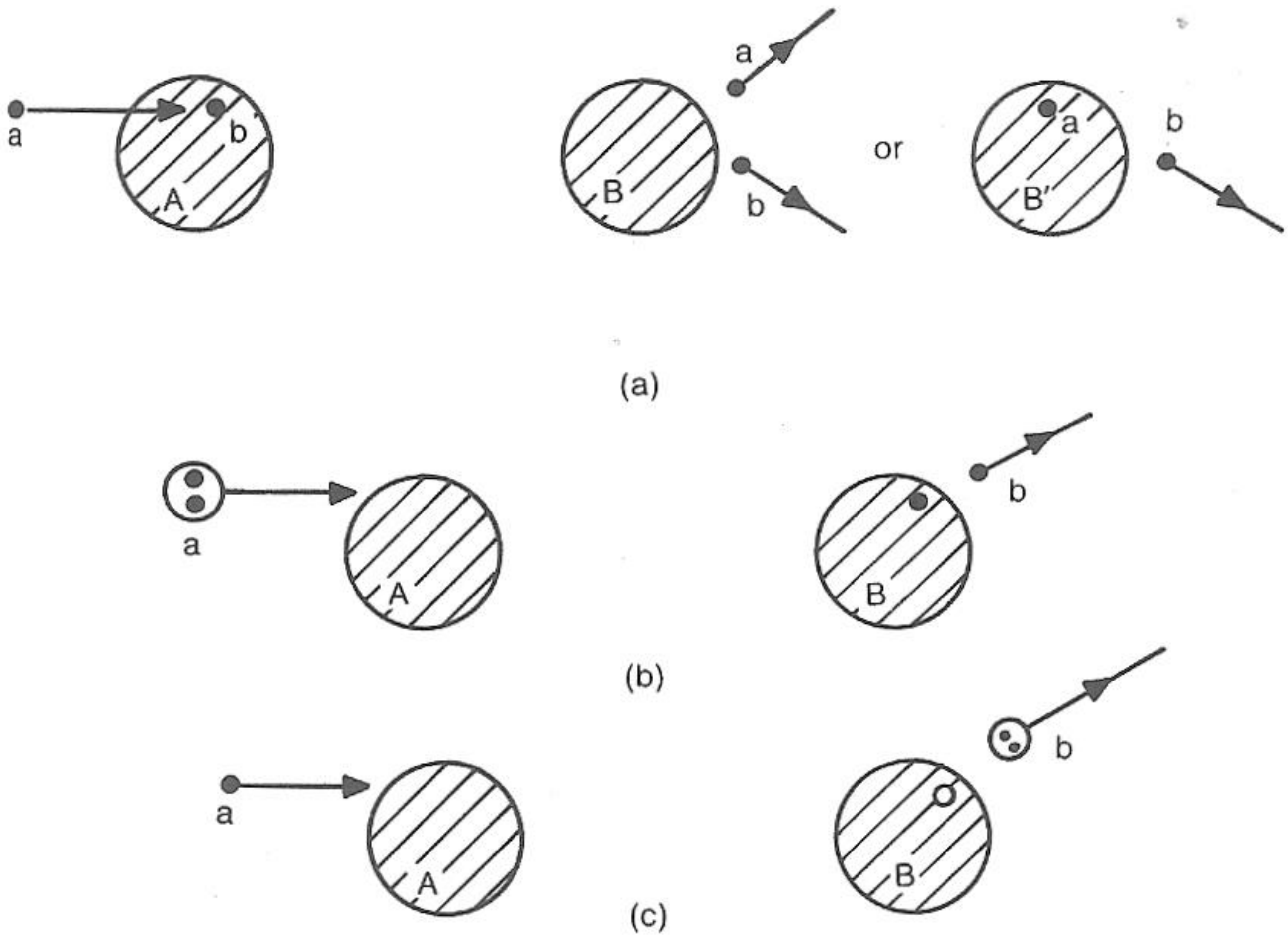
$$a + \mathrm{A} \rightarrow a + \mathrm{A}^*$$

… both examples of **direct reactions** … where the incident particle interacts in a time comparable to the time taken to transit the nucleus.

Other direct reactions …

pick-up reactions: e.g. $p + {}^{16}\mathrm{O} \rightarrow d + {}^{15}\mathrm{O}$ or ${}^{16}\mathrm{O}(p, d)\,{}^{15}\mathrm{O}$

stripping reactions: e.g. $d + {}^{16}\mathrm{O} \rightarrow p + {}^{17}\mathrm{O}$ or ${}^{16}\mathrm{O}(d, p)\,{}^{17}\mathrm{O}$



“knock-out” reaction

“stripping” reaction

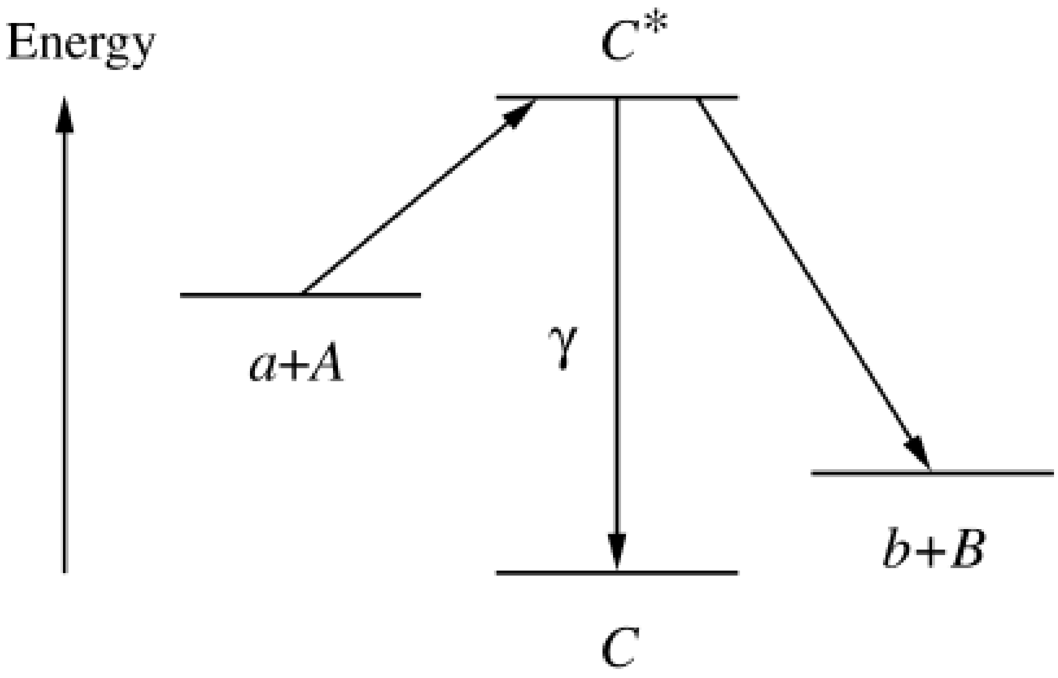
“pick-up” reaction

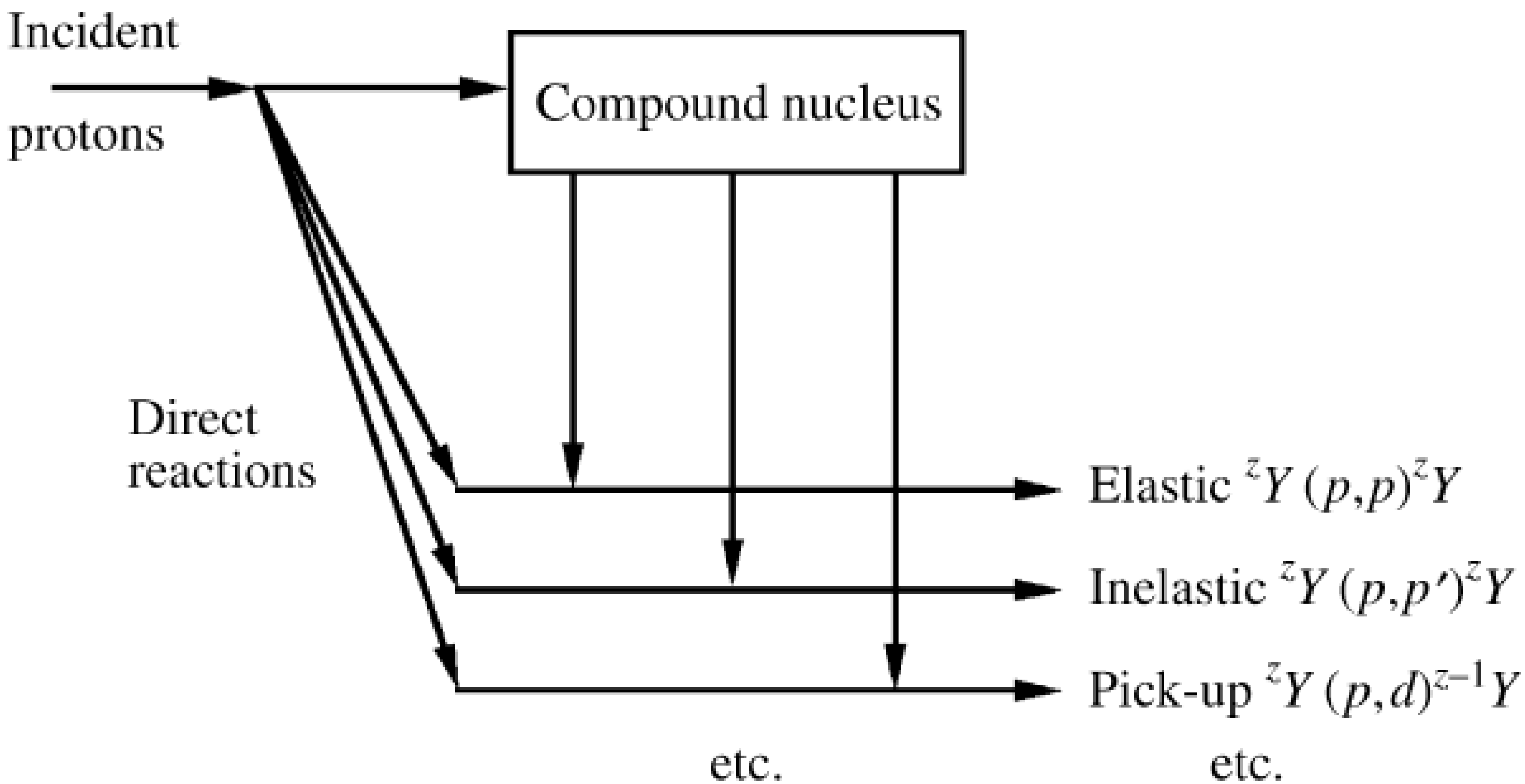
Second important class … **compound nucleus reactions** … when the projectile becomes loosely bound in the nucleus in an intermediate state and shares all its energy with all the other nuclear constituents. Compound nucleus will decay by emitting particles or gamma rays … often many **channels** possible.

$$a + \mathrm{A} \rightarrow \mathrm{C}^* \rightarrow b + \mathrm{B}$$

or

$$a + \mathrm{A} \rightarrow \mathrm{C}^* \rightarrow \mathrm{C} + \gamma$$




Incident
protons
Compound nucleus
Direct
reactions
Elastic ${}^{z}Y\,(p,p)^{z}Y$
Inelastic ${}^{z}Y\,(p,p')^{z}Y$
Pick-up ${}^{z}Y\,(p,d)^{z-1}Y$
etc.
etc.

Unstable states

In the case of an unstable state, for decays at rest, measure the **lifetime at rest** τ , or the **natural decay width** $\Gamma = \hbar/\tau$

… which is a measure of the rate of the decay reaction.

In general, the **total decay width** $\Gamma = \sum_f \Gamma_f$

where Γ_f is the partial width for final state *f*.

Then the **branching ratio** for the decay to the state *f*: $B \equiv \frac{\Gamma_f}{\Gamma}$

Resonances

An unstable state of mass M decaying to products of invariant mass W, has a **Breit-Wigner** form

$$N_f(W) \propto \frac{\Gamma_f}{(W-M)^2 c^4 + \Gamma^2/4}$$

…with peak at $W = M$

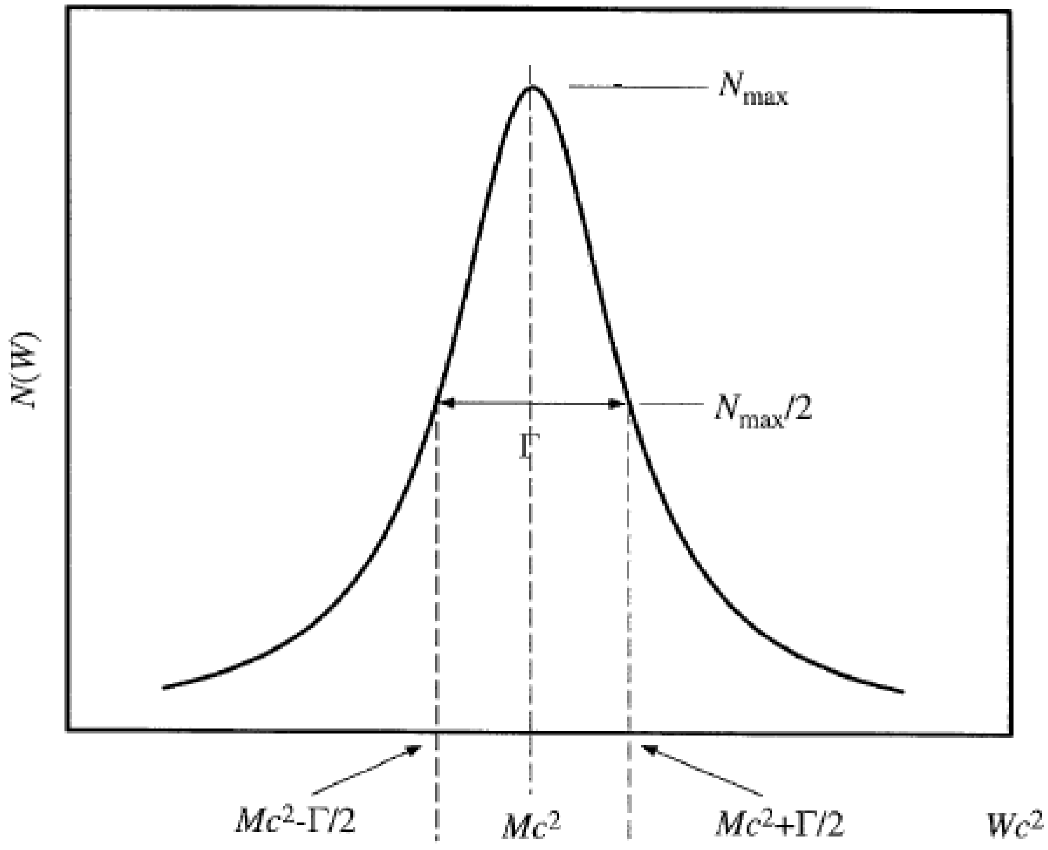
…and half width $\Gamma/2$

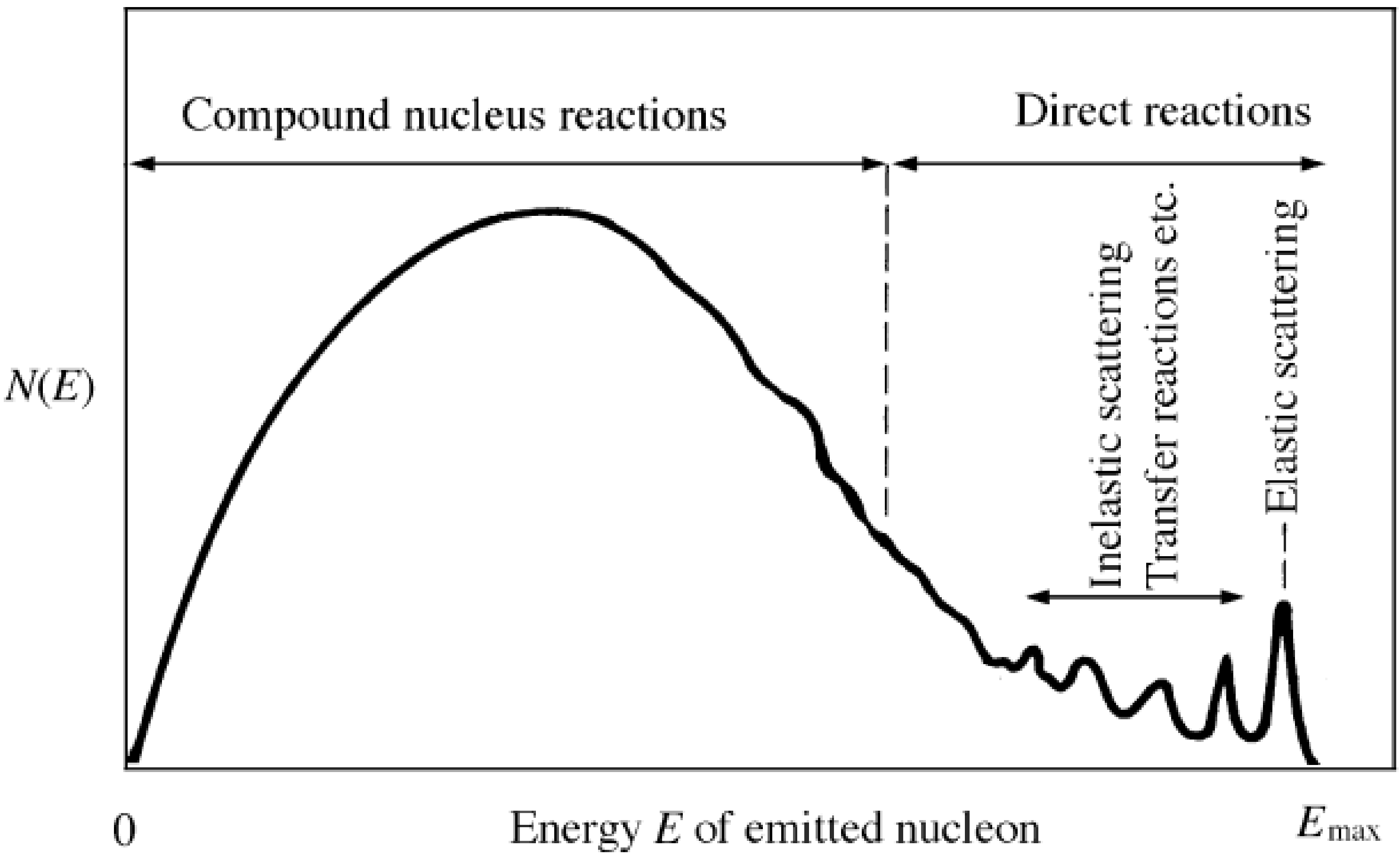
We have produced a **resonance state** ...

… and the cross-section for the reaction has the form

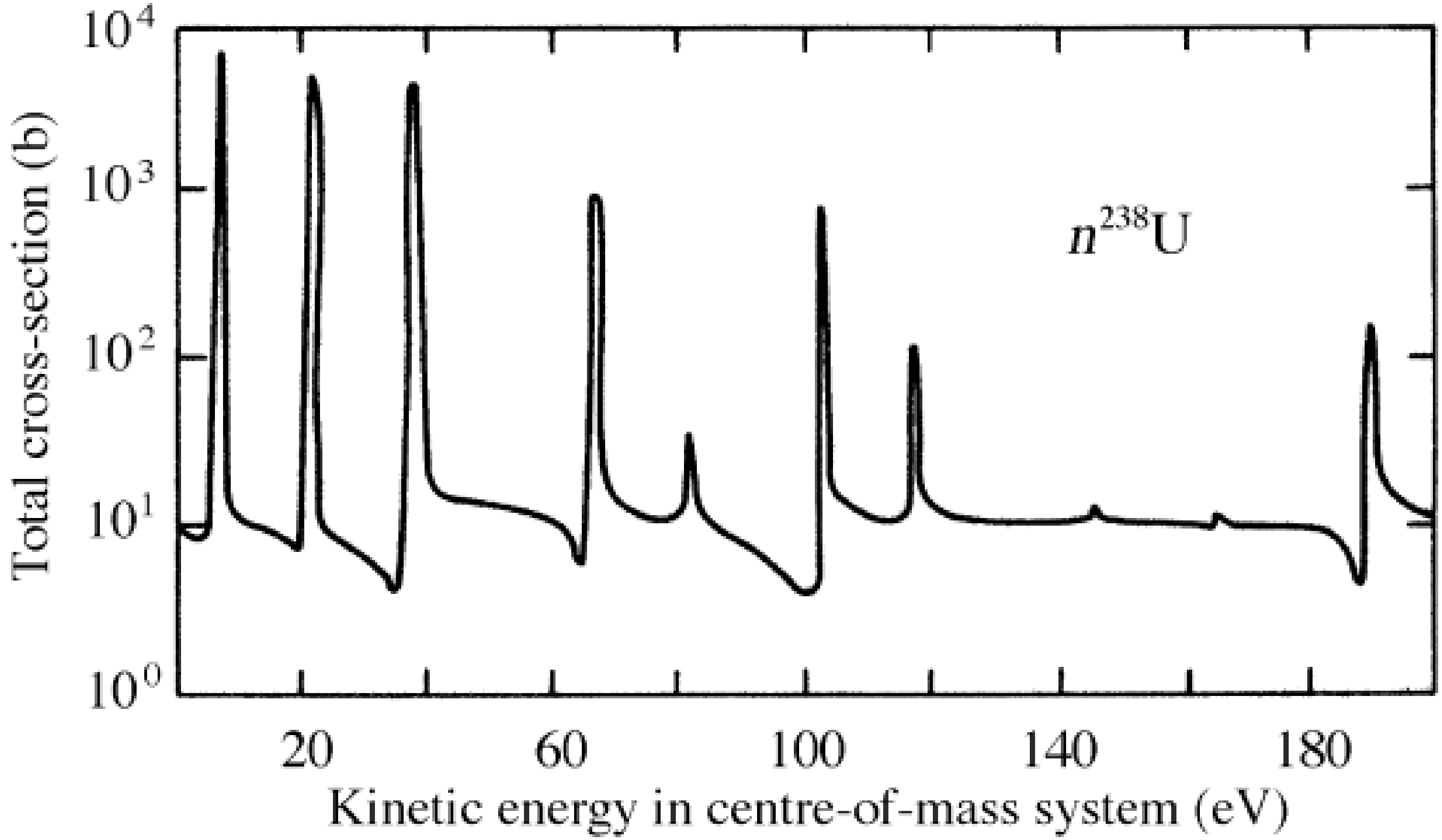
$$\sigma_{if} \propto \frac{\Gamma_i \Gamma_f}{\left(E - Mc^2\right)^2 + \Gamma^2/4}$$

where E is the total energy of the system.





Typical spectrum of energies of the nucleons emitted at a fixed angle in inelastic nucleon-nucleus reactions.



Total cross section for neutron interactions with ^{238}U …
showing many very narrow resonances (with intrinsic widths of the order of 10^{-2} eV) corresponding to excited states of ^{239}U.