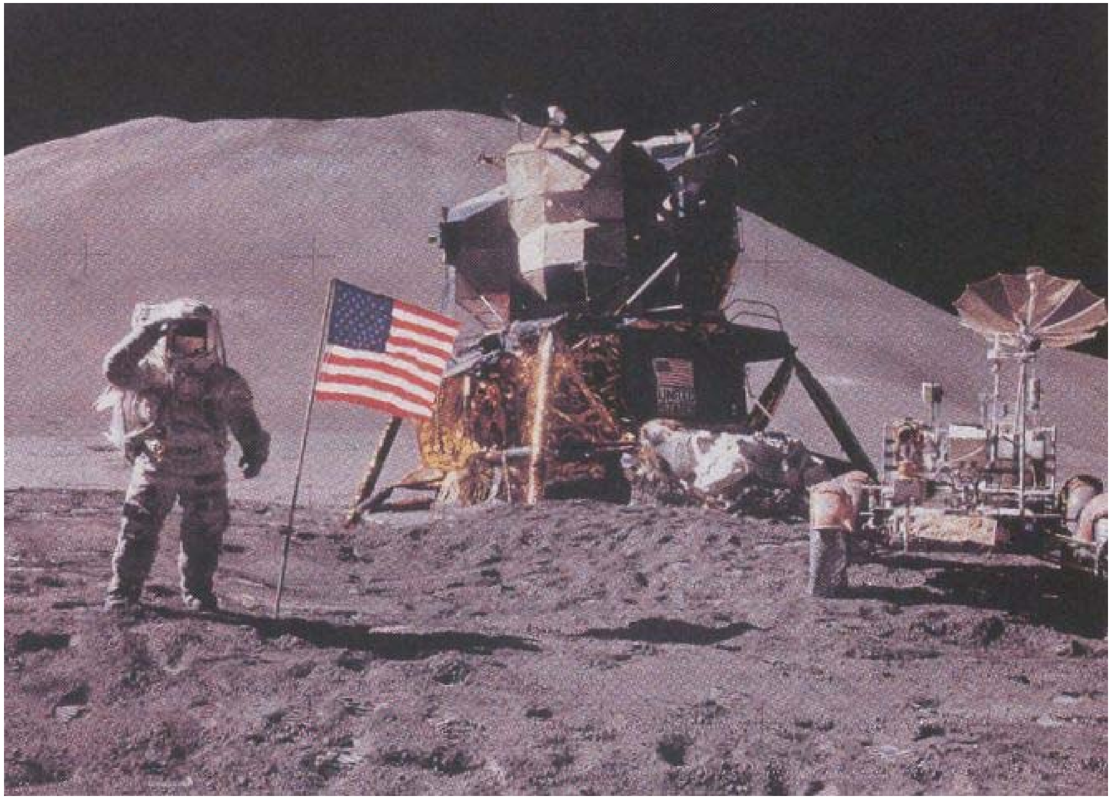

PHY123H

Mechanics Part B

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... see Chapters 5 & 6 in
University Physics
by Ronald Reese

Linear dynamics

... what causes acceleration?

Some observations:

What is the direction of the acceleration of the block in each case?

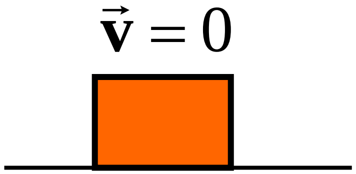
(a) a block traveling on a frictionless surface:

(b) a block traveling on a surface with friction:

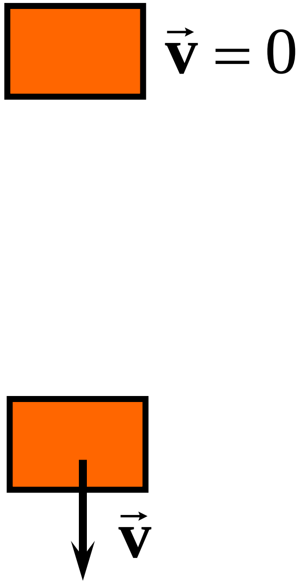
Some more observations:

What is the direction of the acceleration of the block in each case?

(c) a block resting on the floor.

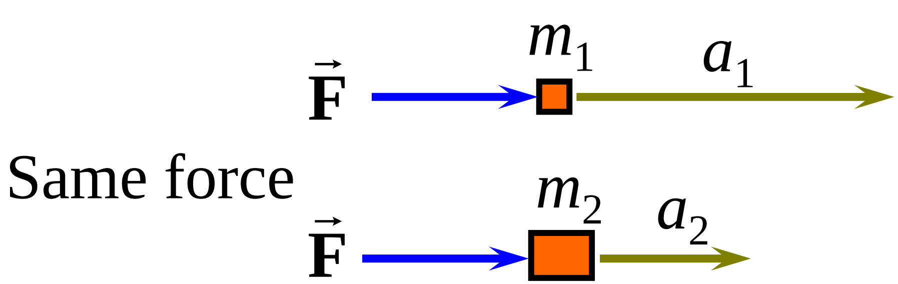


(d) a block dropped from a height above the floor



Newton’s First Law

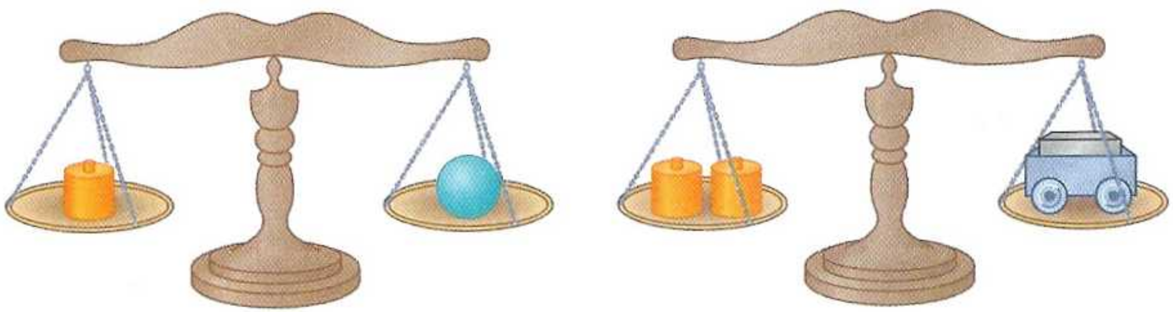
Consider a body on which no force acts.
If the body is at rest, then it will remain at rest. If the body is moving, then it will continue to move with a constant velocity.



$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

... so what is “mass”?

Gravitational mass is determined by balancing gravitational forces ... can use either a mechanical balance or an electronic balance.



Inertial mass of a system is the constant of proportionality between acceleration and the force that causes it.

The inertial mass of an object tells us how much it resists acceleration, while the gravitational mass is a measure of how hard the Earth pulls on the object ... shown by experiment to agree to one part in 10^{12} ... we will simply refer to “mass”.

Newton's Second Law

Newton described force as the rate of change of momentum $\vec{\mathbf{p}}$, where $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

$$\therefore \vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} = \frac{d}{dt}\left(m\vec{\mathbf{v}}\right) = \vec{\mathbf{v}}\frac{dm}{dt} + m\frac{d\vec{\mathbf{v}}}{dt}$$

In the special case of constant mass $\frac{dm}{dt} = 0$

$$\therefore \quad \vec{\mathbf{F}} = m\frac{d\vec{\mathbf{v}}}{dt} = m\vec{\mathbf{a}}$$

Newton's Second Law for a single force: When a single force acts on a object, it will cause the object to accelerate in the direction of the force. The magnitude of the acceleration is the ratio of the magnitude of the force to the mass of the object.

A force of 1 newton gives a mass of 1 kg an acceleration of 1 m s^{-2}.

NI is contained in NII: If $\vec{\mathbf{F}} = 0$, then $\vec{\mathbf{a}} = 0$.

Figuring Physics

An object moves toward the left at decreasing speed.

The direction of the <u>resultant force</u> acting on the object is:

(A) toward the left
(B) zero
(C) toward the right

If we have multiple forces on a body (acting in three dimensions) then the resultant force is the vector sum of all the forces acting on the body.

$$\vec{\mathbf{F}}_{\text{resultant}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \ldots$$

Newton's First Law may then be stated as: Consider a body on which no resultant force acts, i.e. $\vec{\mathbf{F}}_{\text{resultant}} = 0$. If the body is at rest, then it will remain at rest. If the body is moving, then it will continue to move with a constant velocity.

... and **Newton's Second Law**: The acceleration of a body is given by the resultant force acting on the body divided by its mass.

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}_{\text{resultant}}}{m}$$

Figuring Physics

Object one moves on horizontal, frictionless surface at a constant velocity of magnitude 10 m s^{-1}.
Object two of the same mass moves on the same surface at a constant velocity of magnitude 20 m s^{-1}.
Compare the resultant force acting on each object.

(A) $\vec{\mathbf{F}}_{\text{resultant}}$ (on 1) $\neq$ $\vec{\mathbf{F}}_{\text{resultant}}$ (on 2)

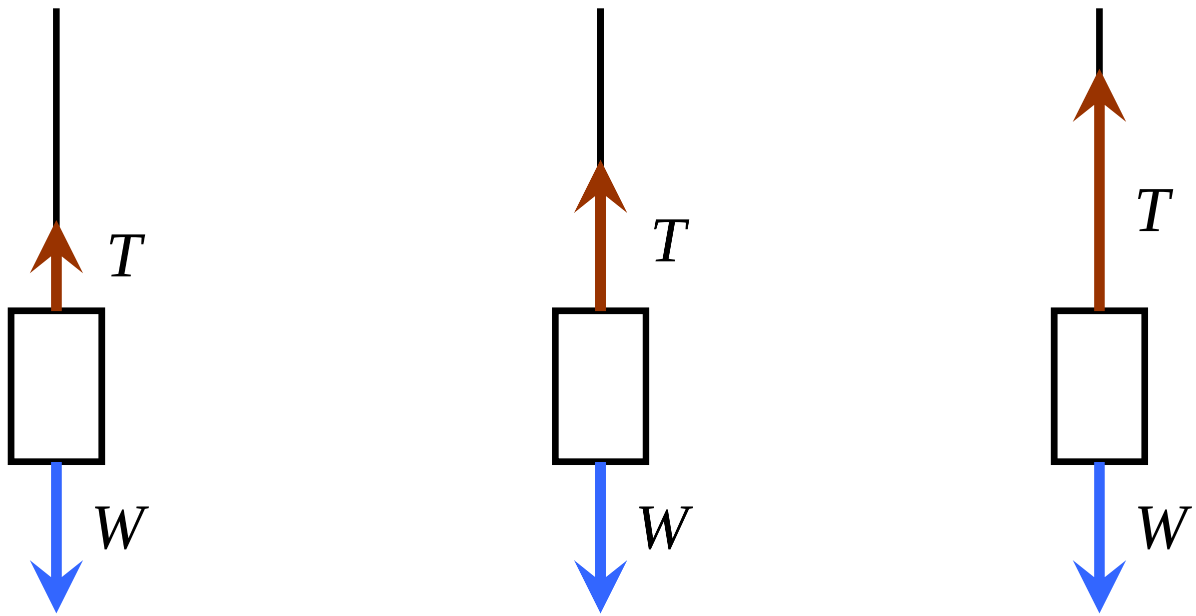
(B) $\vec{\mathbf{F}}_{\text{resultant}}$ (on 1) $=$ $\vec{\mathbf{F}}_{\text{resultant}}$ (on 2) $\neq 0$

(C) $\vec{\mathbf{F}}_{\text{resultant}}$ (on 1) $=$ $\vec{\mathbf{F}}_{\text{resultant}}$ (on 2) $= 0$

Figuring Physics

An elevator moves up at decreasing speed.
Which diagram below best represents the forces acting on the elevator?

(A) (B) (C)



Newton’s Third Law

When two objects exert forces on each other, the force that A exerts on B has the same magnitude, but opposite direction, to the force that B exerts on A. (“For every action, there is an equal, but opposite, reaction.”)

... more on this later ...

All forces result from an interaction ...
... whether they are "non-contact" forces or "contact forces"

The four fundamental forces of nature: **gravitation**, **electromagnetic**, **strong nuclear** and **weak nuclear** may be referred to as "non-contact" since they can act over empty space.

All other forces (tension in a rope, friction, etc ...) require two bodies to be in **contact** with each other.

In fact, all contact forces can be thought of as the superposition of many small electromagnetic forces between the electrons and protons that the materials "in contact" are made of. Thus contact forces are electromagnetic forces.

The four fundamental forces of nature.

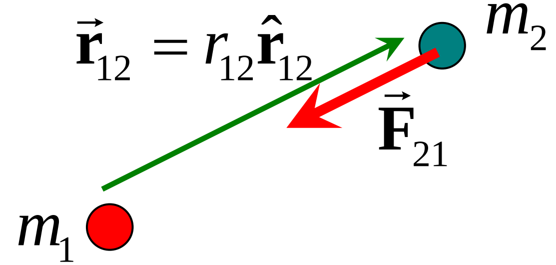
1. Newton's Law of Universal Gravitation.

Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. The force acts along the line joining the two particles.

$$\vec{\mathbf{F}}_{21} = G\frac{m_1 m_2}{r_{12}^2}(-\hat{\mathbf{r}}_{12})$$

force on 2 due to 1

$$\vec{\mathbf{r}}_{12} = r_{12}\hat{\mathbf{r}}_{12}$$



G : Universal gravitational constant.

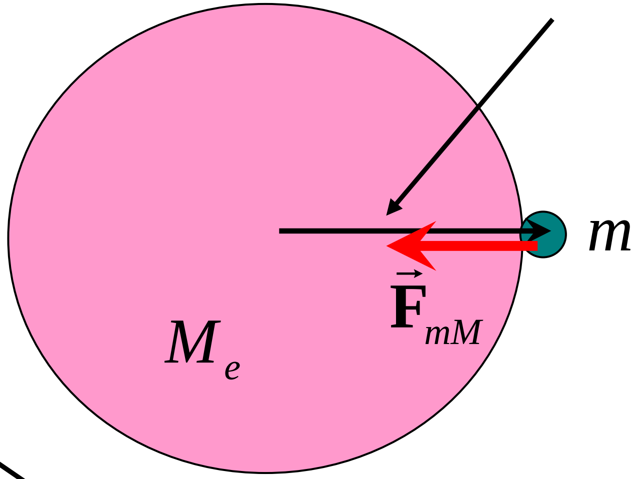
... measured to be 6.672×10^{-11} N m^2 kg^{-2}

By Newton III: $\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$ ... see later

If one of the objects is the earth ...

$$\vec{\mathbf{F}}_{mM} = G\frac{mM_e}{R_e^2}(-\hat{\mathbf{r}}_{Mm})$$

$r_{Mm} \simeq R_e$ (radius of the earth)



By Newton II: $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$

$$\therefore \vec{\mathbf{F}}_{mM} = G\frac{mM_e}{R_e^2}(-\hat{\mathbf{r}}_{Mm}) = m\vec{\mathbf{a}}$$

... introduce the "local gravitation strength" $\vec{\mathbf{g}}$

where $g = G\dfrac{M_e}{R_e^2}$ = 9.80 m s^{-2} in Cape Town

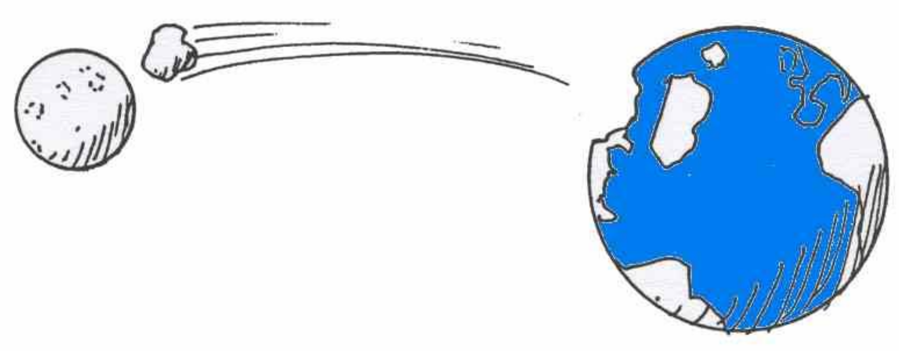
and write $\vec{\mathbf{F}}_{grav} = \vec{\mathbf{W}} = m\vec{\mathbf{g}}$

W is called the "weight" of the object.

Figuring Physics

The gravitational force between the earth and the moon is about 4.7×10^{22} newtons. If, somehow, part of the earth were suddenly transferred to the moon, then the gravitational force between the earth and the moon would

(A) increase
(B) decrease
(C) remain the same



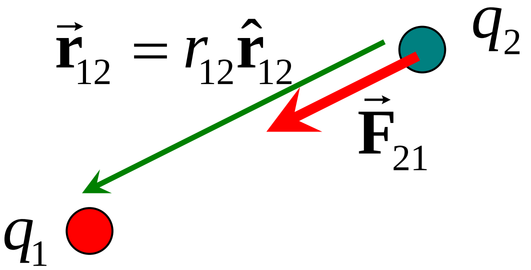
2. Electromagnetic force

... is a combination of electrical and magnetic forces.

For stationary charges, the forces between charges are described by Coulomb's Law:

$$\vec{\mathbf{F}}_{21} = k\frac{q_1 q_2}{r_{12}^2}(\hat{\mathbf{r}}_{12})$$

force on 2
due to 1



k : "Coulomb constant ... measured to be 9.0×10^9 N m^2 C^{-2}

We also know that "like" charges repel each other, and "unlike" charges attract, therefore the Coulomb force can be attractive or repulsive, while the gravitational force is attractive only.

There is a similar description for magnetic forces ... "like" magnetic poles repel, while "unlike" poles (N and S) attract.

3. The strong force.

Binds together the constituents of an atomic nucleus.

4. The weak force

... involved in certain types of radioactive decay

Physicists are presently trying to formulate a unified theory for all the four fundamental forces.

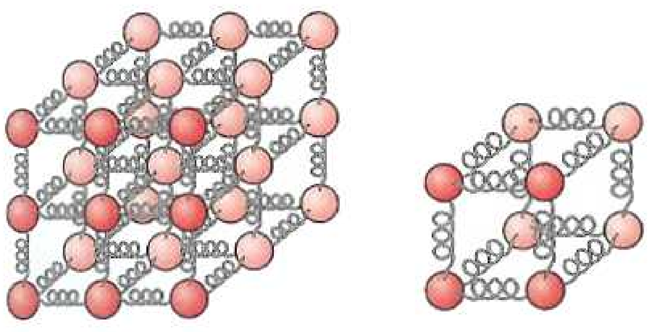
... find out about Grand Unification Theories, Supersymmetry and String Theory ...

Understanding contact forces

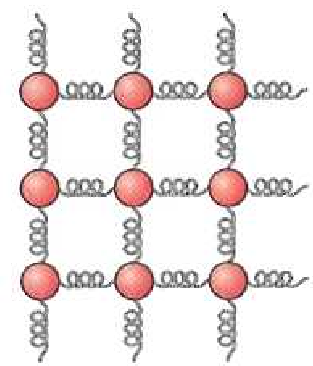
An idealized model of a solid ...

It is useful in this context to visualize matter as being made up of an array of atoms being held together by forces that behave like very stiff springs.

$L \sim 10^{-10}$ m

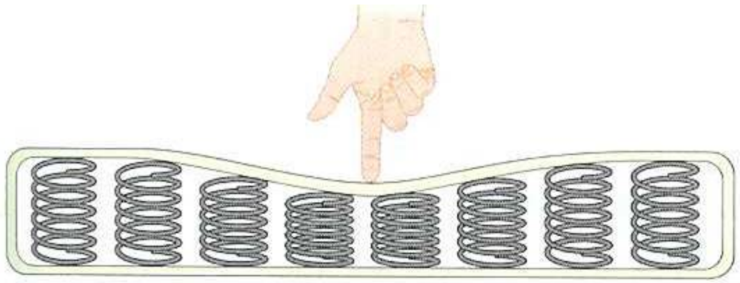


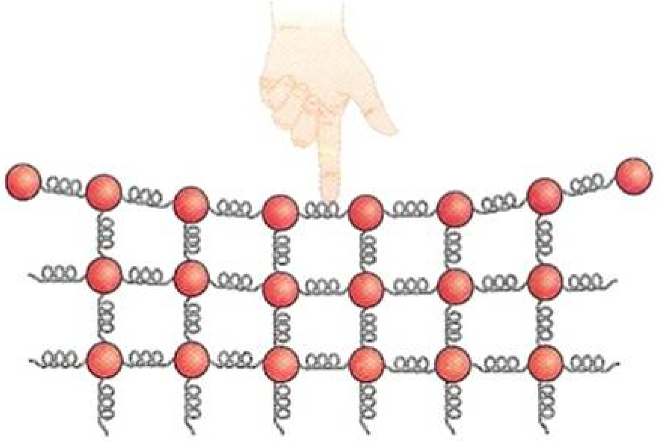
3D



2D

We can think of compressing a solid in the same way as compressing a spring mattress:



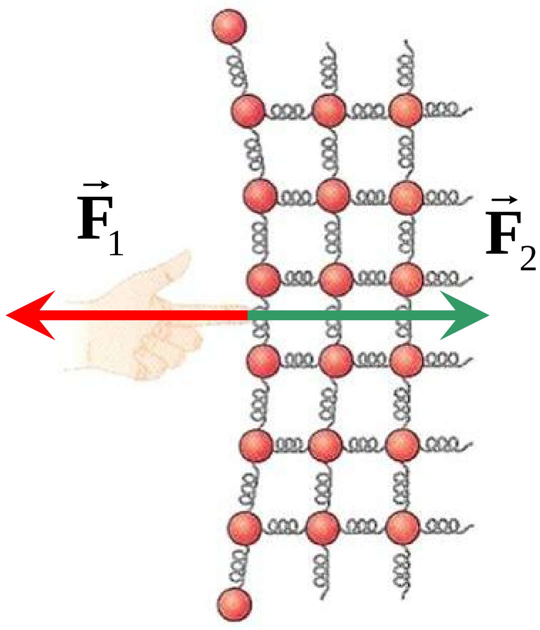


(very much exaggerated)

The harder you press, the more compressed the springs become.

1. The Normal force

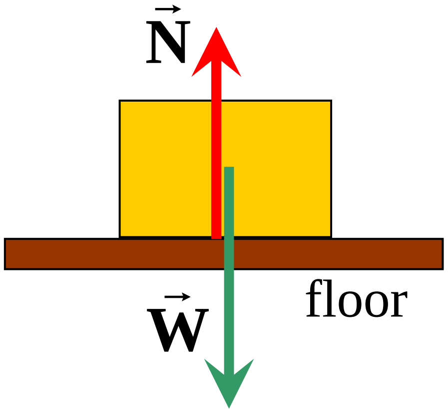
$$\vec{\mathbf{F}}_1 = -\vec{\mathbf{F}}_2$$

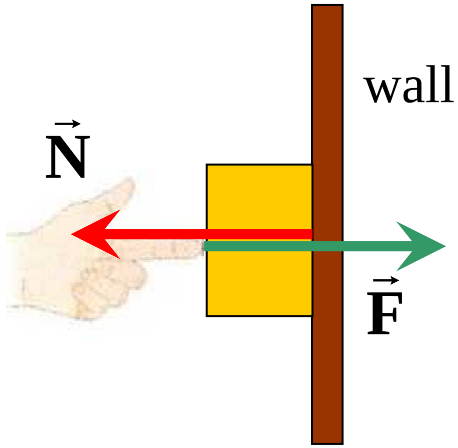


where:

$\vec{\mathbf{F}}_1$ Force exerted by wall on hand

$\vec{\mathbf{F}}_2$ Force exerted by hand on wall

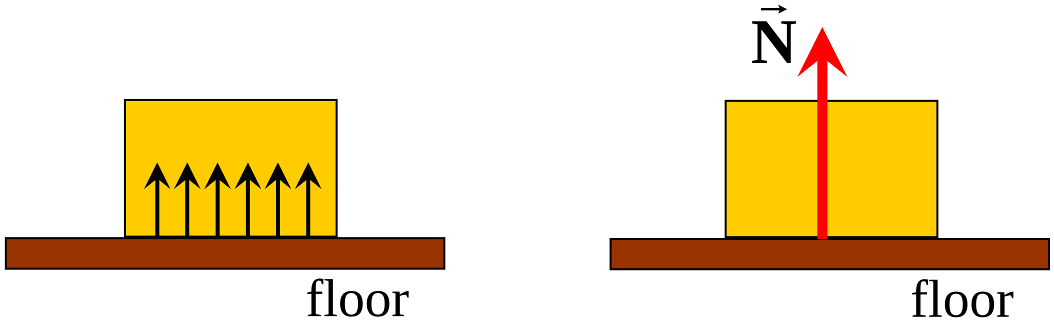




There is a normal force whenever a body is in contact with another body.

... normal force continued...

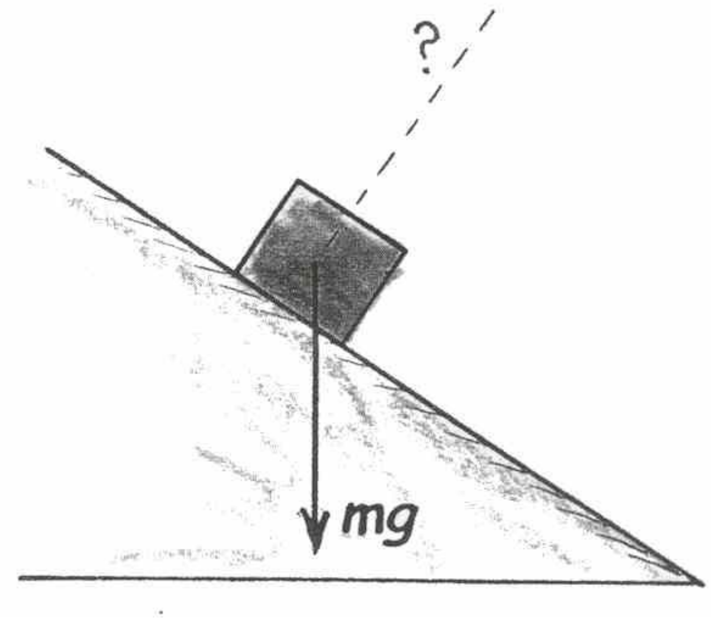
The normal force exerted by a surface on another body is actually the sum of billions of interactions between the surface atoms in the surface and the block. We use a single force vector to summarize all these forces since we are modeling the block as a point particle.



FIGURING PHYSICS

The magnitude of the normal force on a block sliding down a frictionless plane is

(A) equal to *mg*
(B) always greater than *mg*
(C) always less than *mg*



2. Frictional forces

Friction acts parallel to the contact surface and opposes the motion of the object.

From experiment, we find that the frictional force:

- ... is proportional to the normal force
- ... is independent of the area of contact
- ... depends on whether the object is stationary or sliding

Two types of friction: **kinetic friction** and **static friction**

static friction: $f_s = \mu_s N$

kinetic friction: $f_k = \mu_k N$

where μ_s **coefficient of static friction**

μ_k **coefficient of kinetic friction**

$\left.\begin{matrix} \mu_s \\ \mu_k \end{matrix}\right\}$ measured in experiments

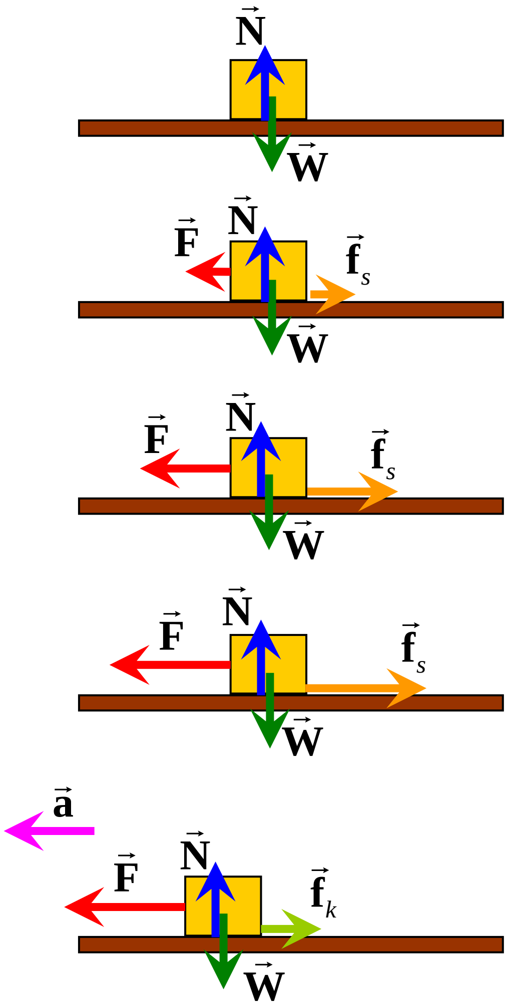
Generally $\mu_s \geq \mu_k$ for a given situation

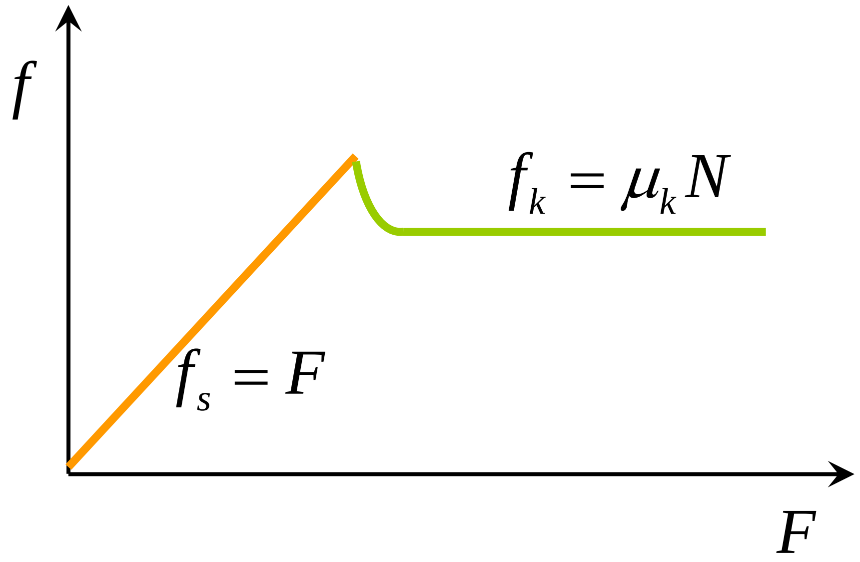
Some approximate coefficients of friction

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminium on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25 - 0.5	0.2
Glass on glass	0.94	0.4
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

Friction continued ...

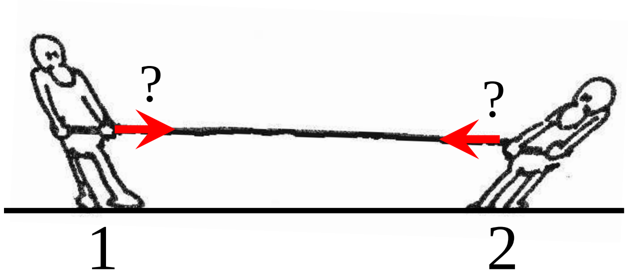
Consider the motion of a block on a surface with friction:



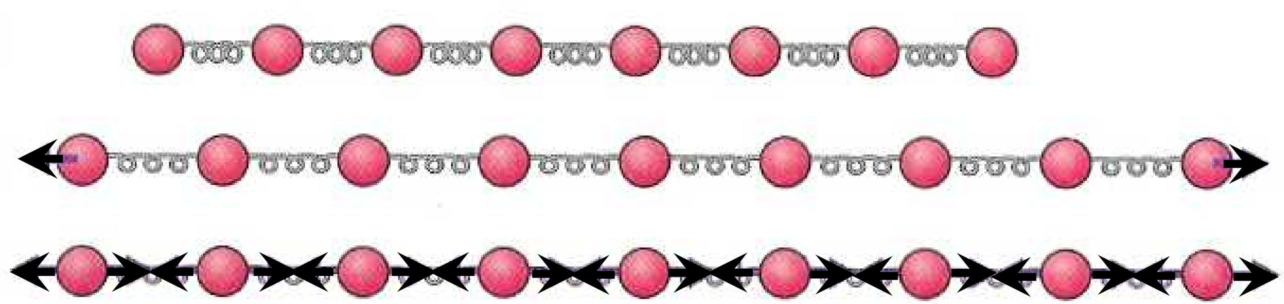


3. Tension

What is the force of Man 1 on Man 2?
What is the force of Man 2 on Man 1?

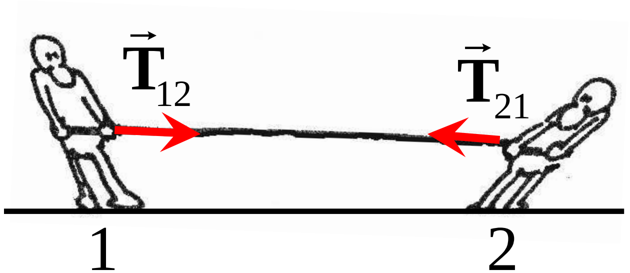


Think of the rope being made up of a long chain of single atoms, each interacting by inter-atomic (spring) forces):



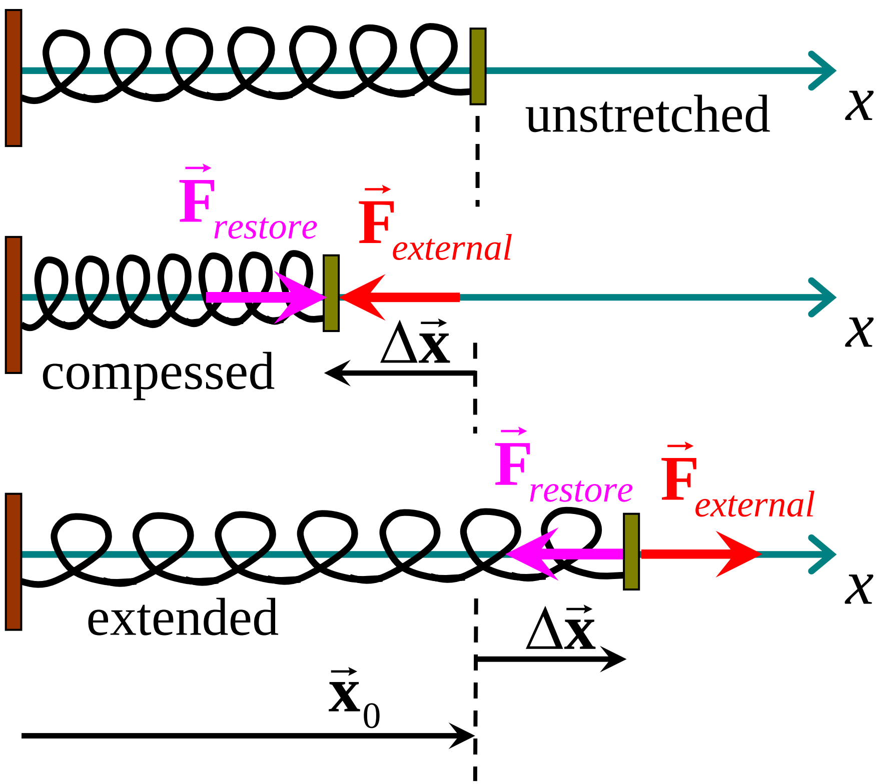
Each atom exerts an equal force on its neighbour, with the resultant force on each individual atom being zero (since the rope is not accelerating).

$$\vec{\mathbf{T}}_{12} = -\vec{\mathbf{T}}_{21}$$



4. Spring forces

The force that a spring exerts is proportional to the distance it is stretched or compressed from its unstretched position.
Since a spring exerts a force opposite to its displacement, it is called a restoring force, and is described by **Hooke's Law**:



$$\vec{\mathbf{F}}_{restore} = -k\Delta\vec{\mathbf{x}}$$

where $\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_0$

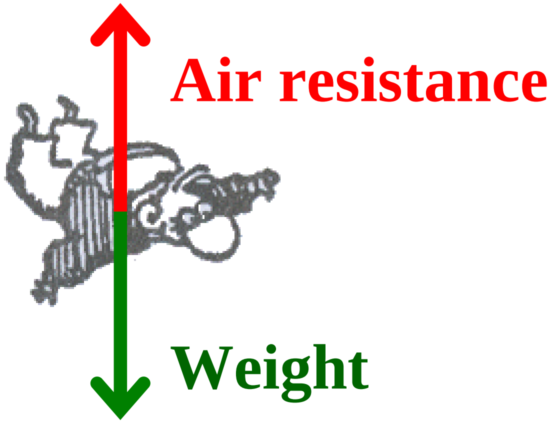
and k is the "spring constant" [N m^{-1}]

$$\vec{\mathbf{F}}_{restore} = -\vec{\mathbf{F}}_{external}$$

4. Drag force and terminal speed

When an object begins to fall through a fluid (such as a skydiver falling through the air), initially the only force acting on the object is gravity, and its acceleration is g.
As the object picks up speed, the frictional or drag force ("air resistance" in air) increases, and the acceleration of the object decreases in magnitude.

Eventually the two forces will be balanced, and the object falls at a constant velocity, called the **terminal velocity**.

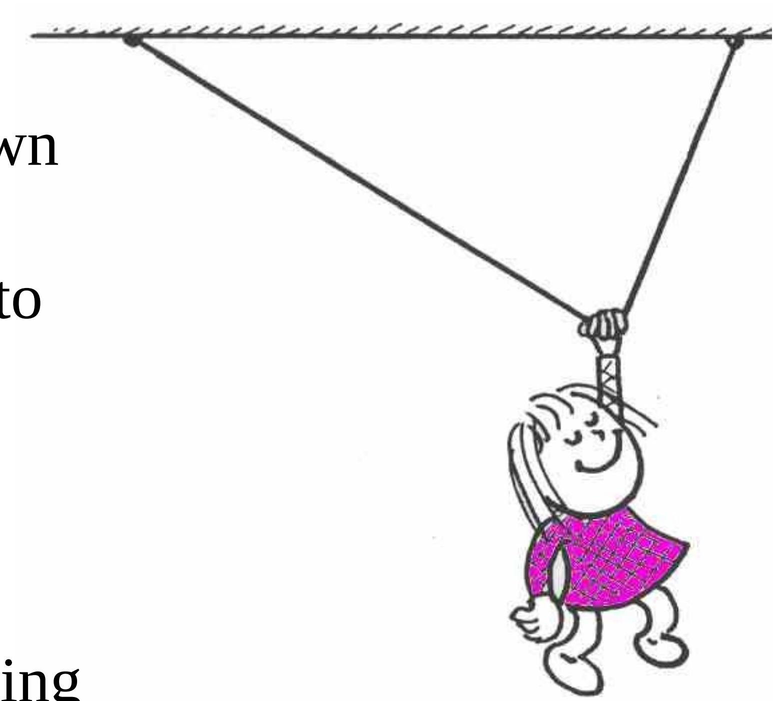


For a skydiver, terminal velocity is between 160 to 240 km h^{-1}.

Figuring Physics

Nellie Newton hangs by one hand motionless from a clothesline as shown - which is on the verge of breaking. Which side of the line is most likely to break?

(A) left side
(B) right side
(C) 50/50 chance of either side breaking



Menu of forces

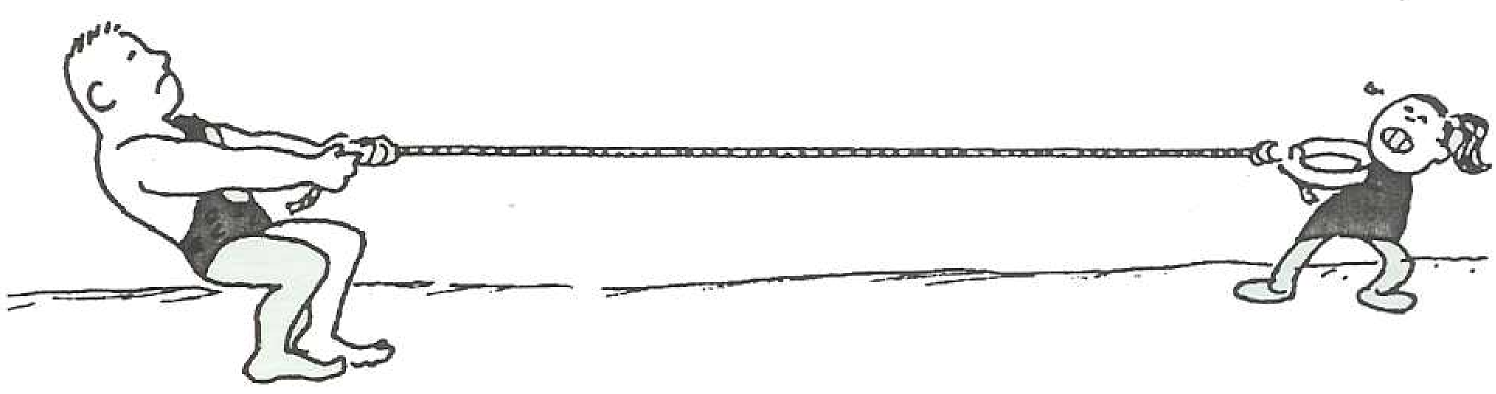
- Gravitational force
- Electromagnetic force
- Strong nuclear force
- Weak nuclear force

- Pushes and pulls
- Normal forces
- Tension in ropes
- Restoring forces in springs
- Friction

Figuring Physics

Arnold Strongman and Suzie Small pull on opposite ends of a rope in a tug of war. The greatest force exerted on the rope is by

(A) Arnold
(B) Suzie
(C) ... both the same.



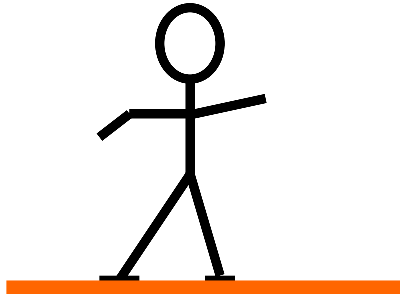
Identifying the significant forces acting on a system

What is a **system**?

... think of drawing an imaginary surface around what you want to investigate ... what is inside the imaginary surface is your **system**.

What do we mean by **significant** forces?

Identify all forces acting on Bob:



Our models will use ideal strings and pulleys (for now):

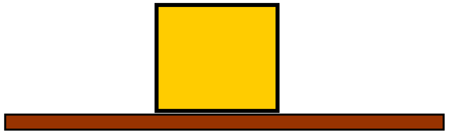
<u>An “ideal” string:</u>
... has negligible mass
... does not stretch when pulled
... can pull but not push
... pulls at any point only in a direction along the line of the string

<u>An “ideal” pulley:</u>
... has negligible mass
... rotates without friction
... allows the rope to run over it without slipping

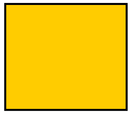
Identifying forces

In each of the situations below, identify all the significant forces acting on the system, as specified.
Draw in each force as a vector arrow and label it.

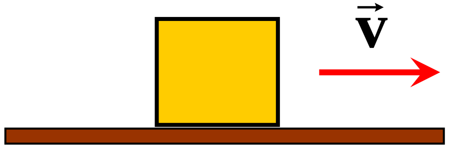
(a)



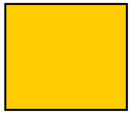
A box is at rest on the floor
System: the box



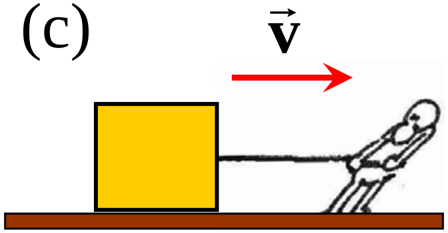
(b)



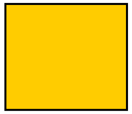
A box is moving at constant velocity on a frictionless surface.
System: the box



(c)

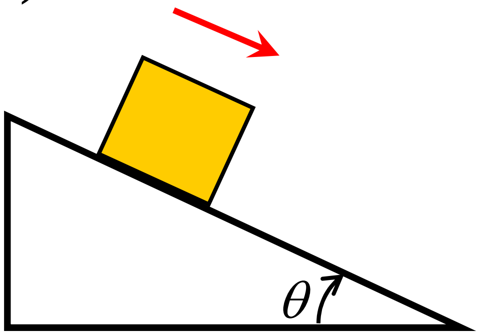


A box is pulled along a floor with friction at constant velocity.
System: the box

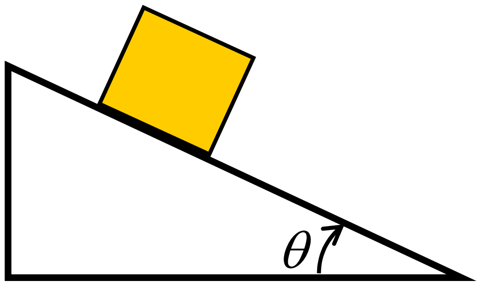


Identifying forces 2 ...

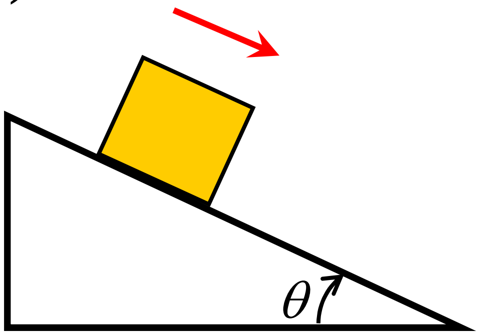
(d)



(e)



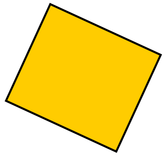
(f)

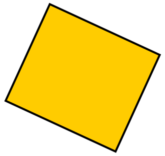


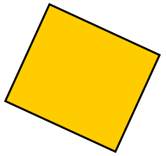
A box slides down a frictionless slope.
System: the box

A box is at rest on a slope.
System: the box

A box slides down a slope with friction.
System: the box

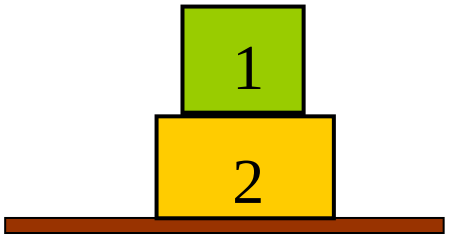






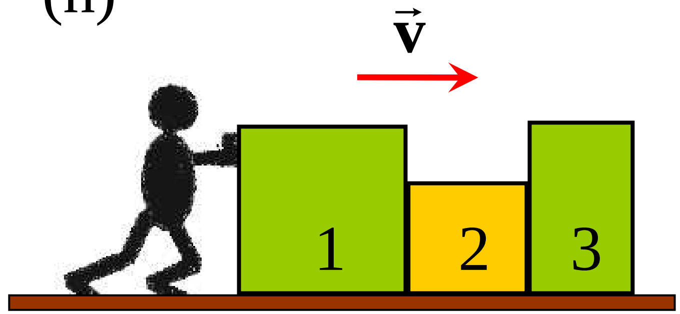
Identifying forces 3 ...

(g)

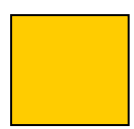


Two blocks are stacked on the floor.
<u>System</u>: box 2

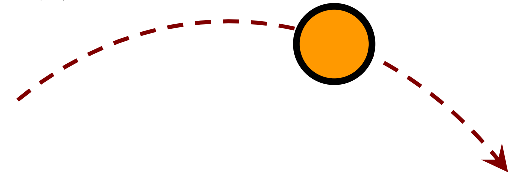
(h)



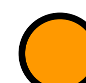
A man pushes 3 blocks across a floor with friction at a constant velocity.
<u>System</u>: box 2



(i)

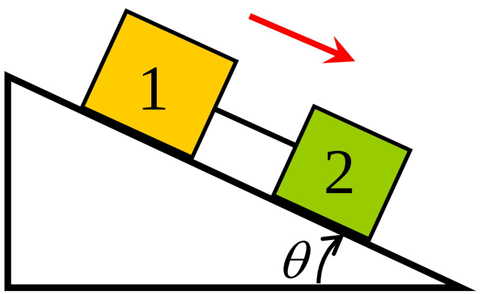


A ball moves through the air in a parabolic path.
<u>System</u>: the ball



Identifying forces 4 ...

(j)

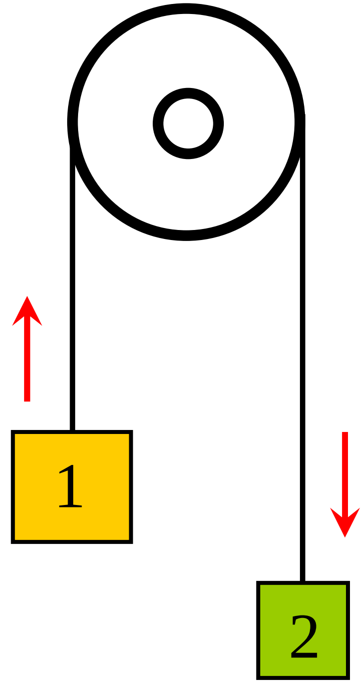


Two boxes connected by a rope slide down a slope with friction.

System 1: box 1

System 2: box 2

(k)

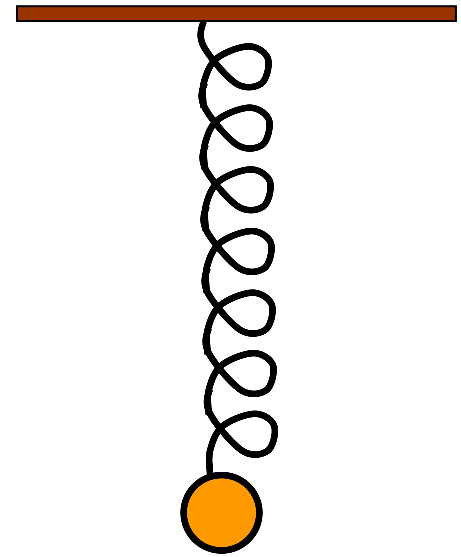


Two boxes connected by a rope slide over a frictionless pulley.

System 1: box 1

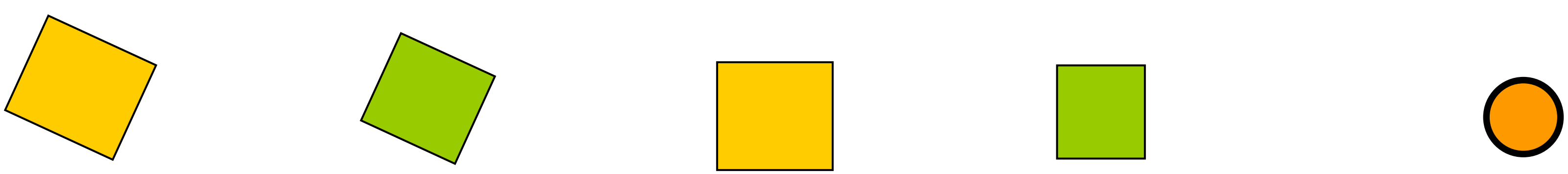
System 2: box 2

(l)



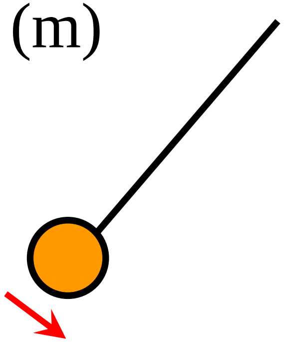
A heavy ball hangs stationary at the end of an extended spring.

System: the ball

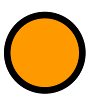


Identifying forces 5 ...

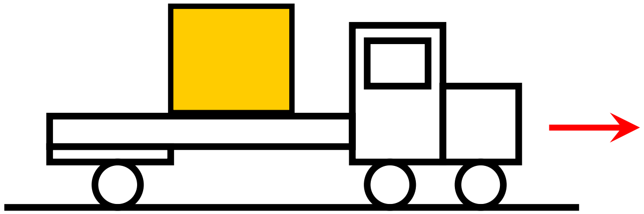
(m)



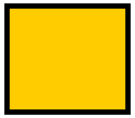
A heavy ball swings at the end of a long string.
System: the ball



(n)



A box sits at the back of a truck which is accelerating towards the right.
System: the box



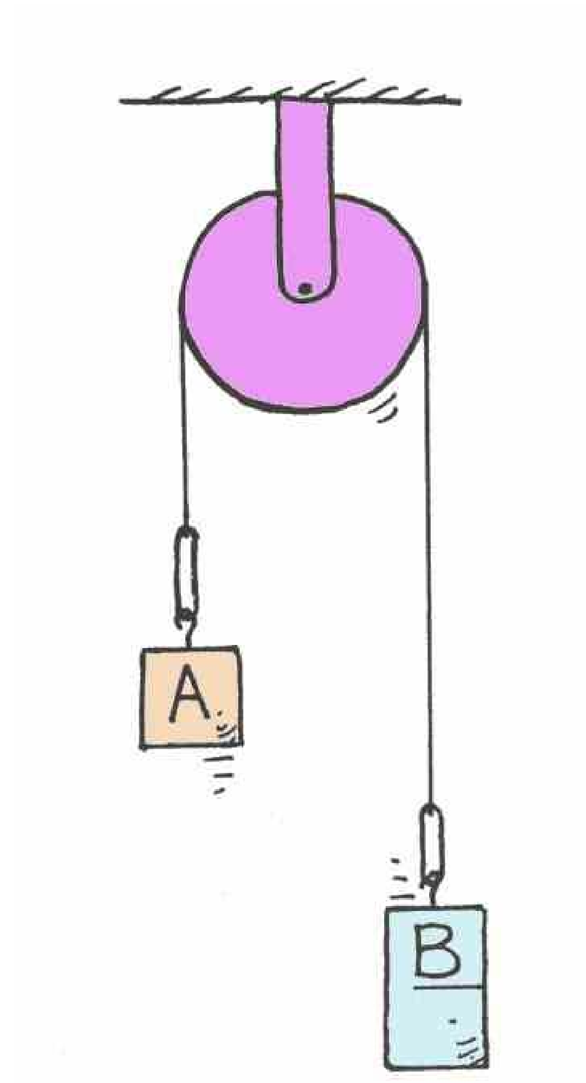
(o)

A walking man.
System: the man

Figuring Physics

Two identical rubber bands connect masses A and B to a string over a light, frictionless pulley. The amount of stretch is greater in the band that connects

(A) mass A
(B) mass B
(C) both the same

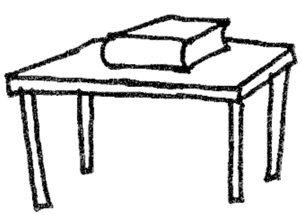


Newton's Third Law

When two objects exert forces on each other, the force that A exerts on B has the same magnitude, but opposite direction, to the force that B exerts on A. ("For every action, there is an equal, but opposite, reaction.")

Identify Newton III force pairs in each of the following scenarios:

(a) A book rests on a table



(b) the moon revolves around the earth

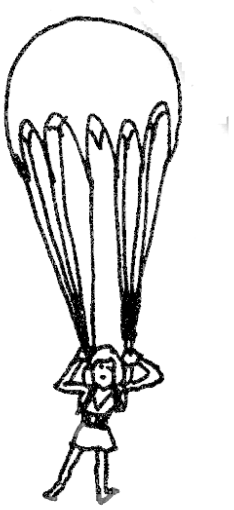
Newton III continued ...

(c) A walking man

(d) Two teams pull on a rope

(e) A man pushes three boxes

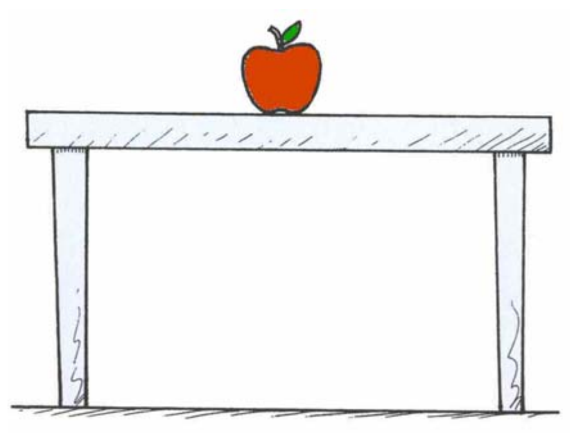
(f) A girl falls with a parachute



FIGURING PHYSICS

Consider the apple at rest on the table. If the call the gravitational force exerted by the earth on the apple **action**, what is the **reaction** force according to Newton's Third Law?

(A) The normal force exerted by the table on the apple
(B) The gravitational force exerted by the apple on the earth.
(C) The gravitational force of the table on the apple.



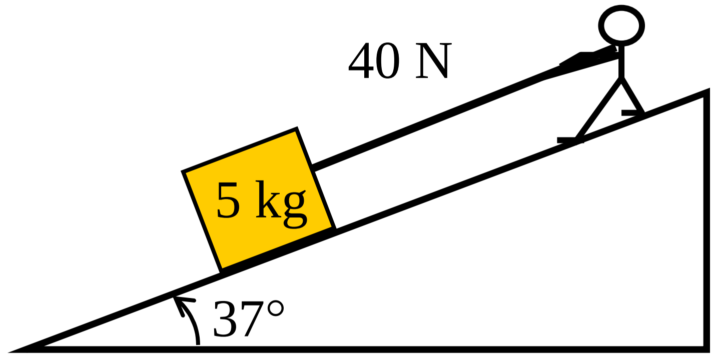
Drawing and using free body diagrams

When approaching problems that require you to add a number of forces acting on a system, it is essential to draw a free body (or force) diagram.
A **free body diagram** uses arrows to represent all forces acting on an object or objects in a system. The tail of an arrow representing a force is placed at the point where the force acts on the object and the arrow points in the direction of the force.

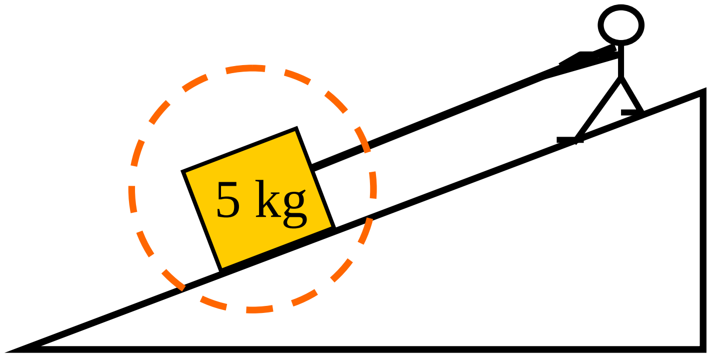
Consider the following problem as an example:

A 5 kg box is being pulled up a frictionless 37° slope by a rope parallel to the slope. If the tension in the rope is 40 N , what is the acceleration of the block ?

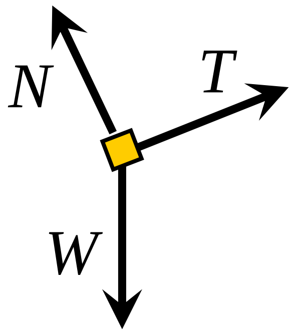
Step 1:
Draw a picture. Include as much information as you can.



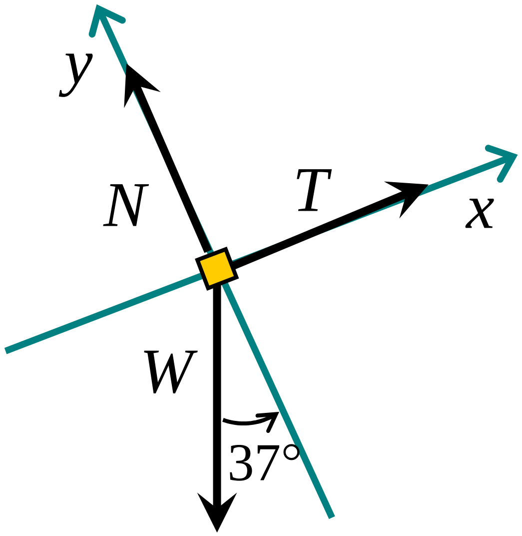
Step 2:
Circle the system on your picture.
The system is the object(s) on which the external forces are acting.
In this case the system is the box.



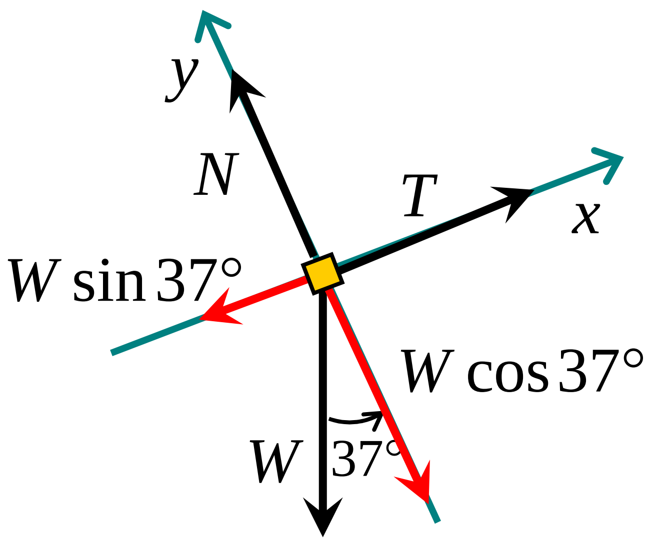
Step 3:
Redraw the system as a blob. Identify all the significant forces acting on the system and label these forces clearly with suitable symbols.
Try to make the relative lengths of the arrows representative of the magnitudes of the forces (this is not always possible).



Step 4:
Choose and draw in a set of convenient coordinate axes. Usually one axis is orientated in the direction of motion and the other axis is orientated perpendicular to the direction of motion. Write in the magnitudes of any angles.

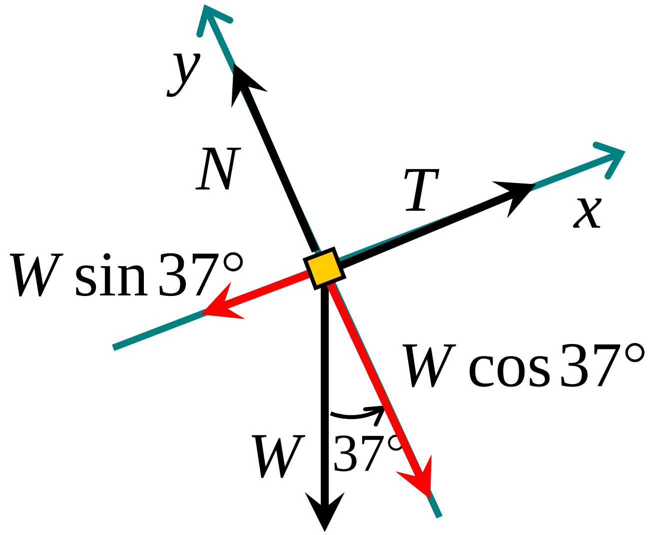


Step 5:
Resolve any forces not falling onto an axis into components along the axes. Draw these components acting on the system.



Step 6:

Now apply Newton's Second Law in component form along each axis:



$$\sum \vec{\mathbf{F}}_x = m\vec{\mathbf{a}}_x$$

$$T\hat{\mathbf{i}} + W\sin 37°(-\hat{\mathbf{i}}) = ma_x\hat{\mathbf{i}}$$

$$T - W\sin 37° = ma_x$$

$$\sum \vec{\mathbf{F}}_y = m\vec{\mathbf{a}}_y$$

$$N\hat{\mathbf{j}} + W\cos 37°(-\hat{\mathbf{j}}) = ma_y\hat{\mathbf{j}}$$

$$N - W\cos 37° = 0$$

... and solve for a_x (and N).

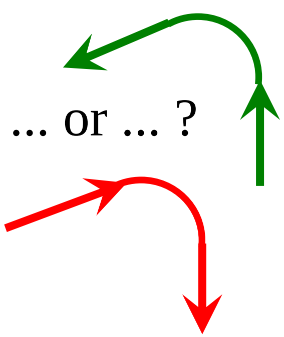
$$(40) - (5)(9.8)\sin 37° = (5)a_x$$

$a_x = 2.6 \text{ m s}^{-2}$ or $\vec{\mathbf{a}}_x = 2.6\,\hat{\mathbf{i}} \text{ m s}^{-2}$

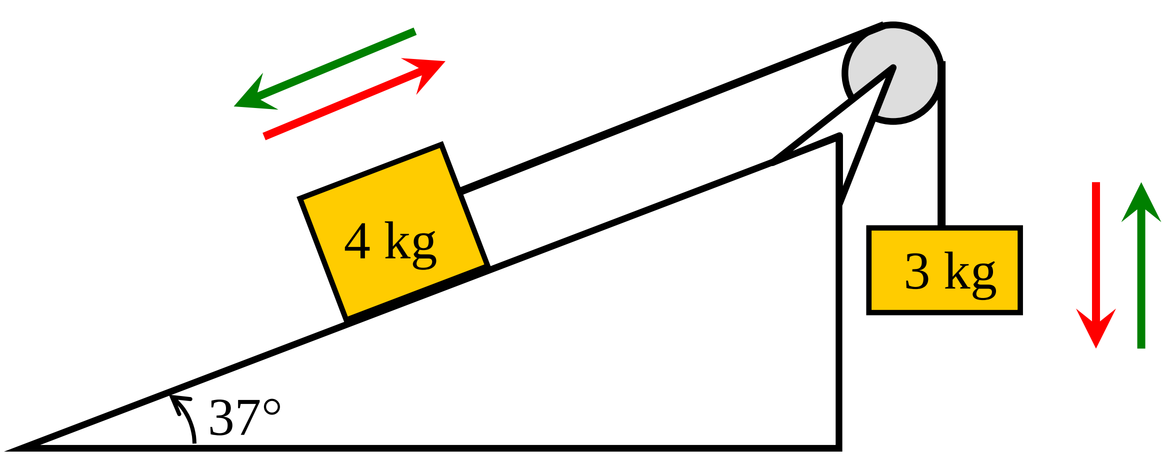
Drawing and using free body diagrams continued

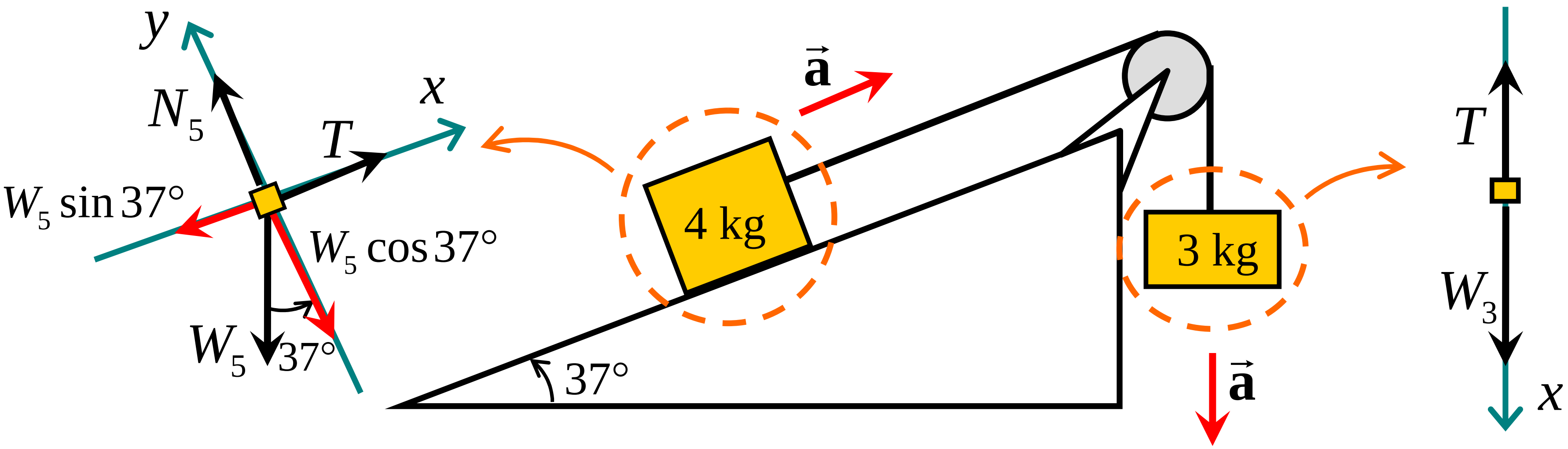
If two or more systems are involved in a problem, then <u>be consistent</u> when you choose the direction of acceleration. It does not matter which way you think the systems will move providing you are consistent with each, bearing in mind that they are connected.

Which way will the masses accelerate ?



... or ... ?





$$\sum \vec{\mathbf{F}}_x = m\vec{\mathbf{a}}_x$$

$$T\hat{\mathbf{i}} + W_5 \sin 37°(-\hat{\mathbf{i}}) = m_5 a_x \hat{\mathbf{i}}$$

$$T - W_5 \sin 37° = m_5 a_x$$

$$T - (4)(9.8)\sin 37° = (4)a_x$$

$$\sum \vec{\mathbf{F}}_x = m\vec{\mathbf{a}}_x$$

$$W_3\hat{\mathbf{i}} + T(-\hat{\mathbf{i}}) = m_3 a_x \hat{\mathbf{i}}$$

$$W_3 - T = m_3 a_x$$

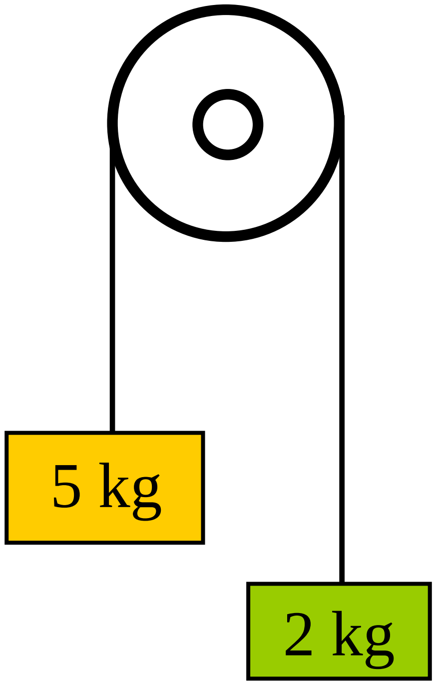
$$(3)(9.8) - T = (3)a_x$$

... and solve for a_x (and T) ...

$a_x = 0.83 \text{ m s}^{-2}$ or $\vec{\mathbf{a}}_x = 0.83\,\hat{\mathbf{i}} \text{ m s}^{-2}$

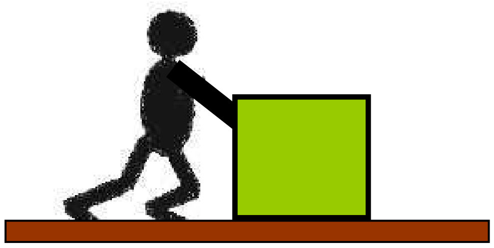
Two masses are connected by a light string over a massless, frictionless pulley.

Draw a free body diagram for each mass.
Apply NII in component form to each system.
Calculate the acceleration of the masses.



Qualitative reasoning about Newtonian processes ...1

A person pushes a crate so that it moves at constant velocity toward the right. A free body diagram for the crate is shown (where the arrows are not necessarily the correct relative lengths. Which choice below best represents the relative magnitude of the forces ?



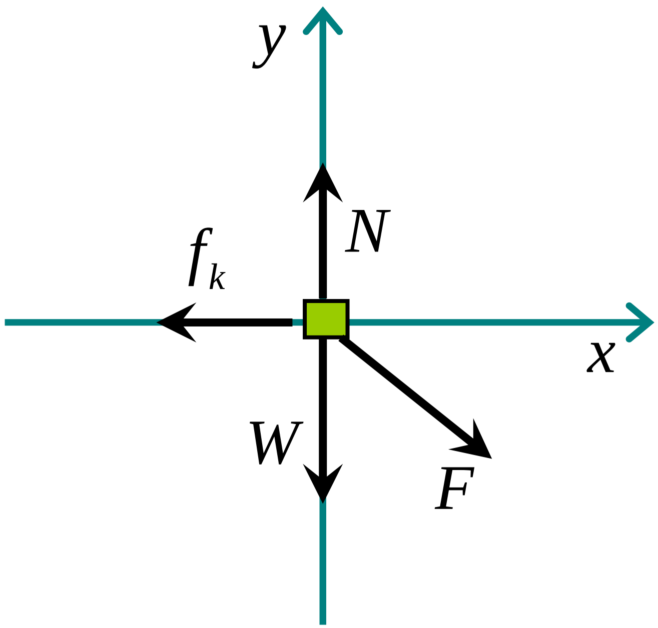
(a) $F = f_k$ and $N = W$

(b) $F = f_k$ and $N > W$

(c) $F = f_k$ and $N < W$

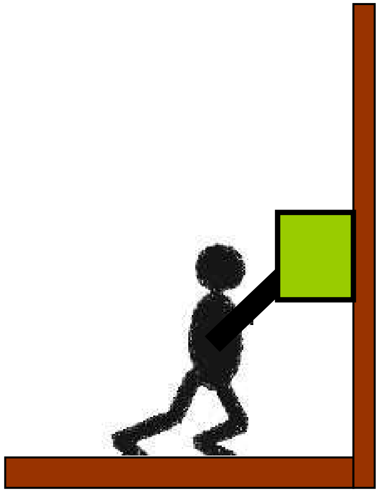
(d) $F > f_k$ and $N = W$

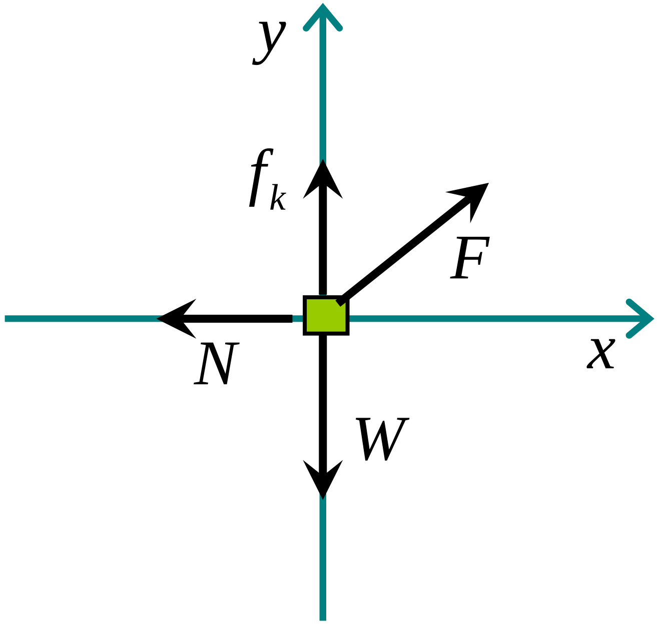
(e) $F > f_k$ and $N > W$



Qualitative reasoning about Newtonian processes ...2

A person pushes a crate as it moves down a vertical wall at constant velocity. A free body diagram for the crate is shown (where the arrows are not necessarily the correct relative lengths. Which choice below best represents the relative magnitude of the forces ?

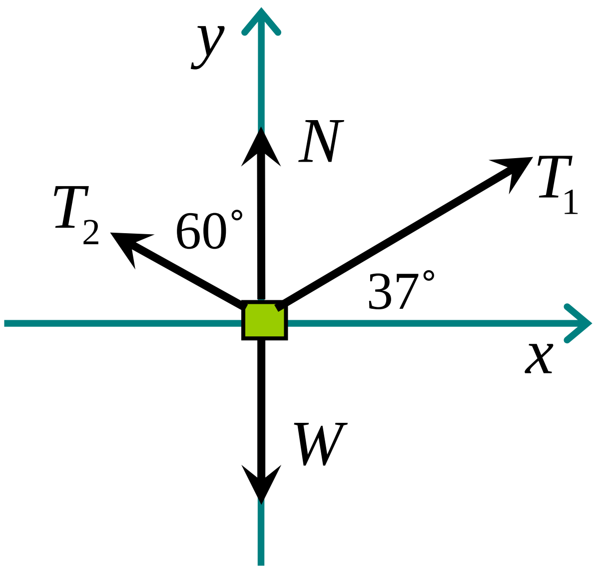




(a) $W = f_k$ and $N = F$
(b) $W = f_k$ and $N > F$
(c) $W = f_k$ and $N < F$
(d) $W > f_k$ and $N = F$
(e) $W > f_k$ and $N > F$

Example 1

For the free body diagram shown, apply NII in component form.

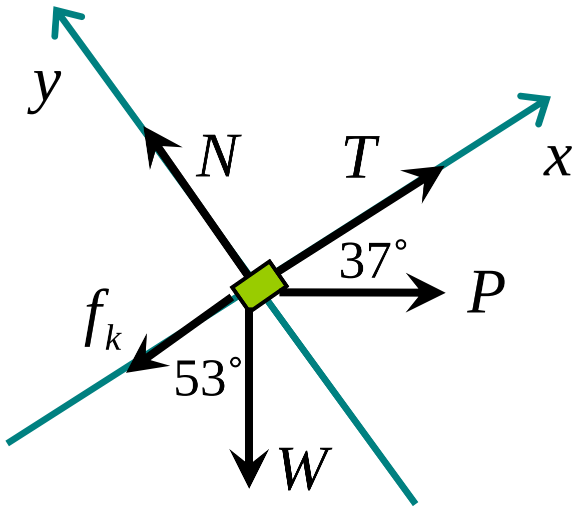


$$\sum \vec{\mathbf{F}}_x = m\vec{\mathbf{a}}_x$$

$$\sum \vec{\mathbf{F}}_y = m\vec{\mathbf{a}}_y$$

Example 2

For the free body diagram shown, apply NII in component form.

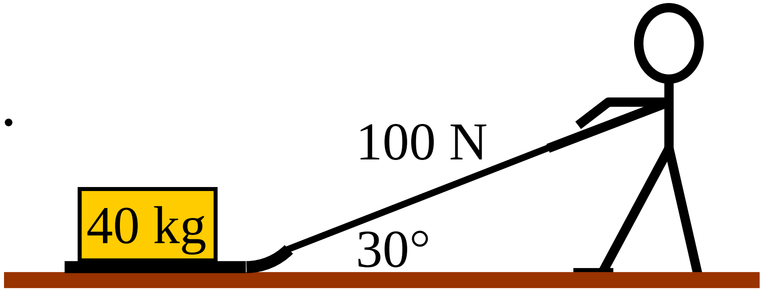


$$\sum \vec{\mathbf{F}}_x = m\vec{\mathbf{a}}_x$$

$$\sum \vec{\mathbf{F}}_y = m\vec{\mathbf{a}}_y$$

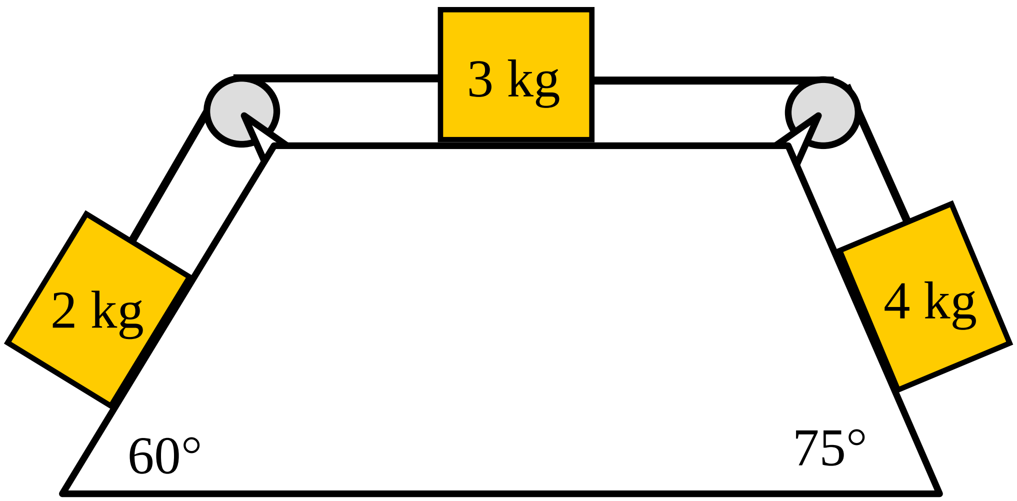
Example 3

A 40 kg sled is pulled to the right at a constant speed on a horizontal surface. The tension in the rope is 100 N. Determine the coefficient of kinetic friction between the sled and the ground.



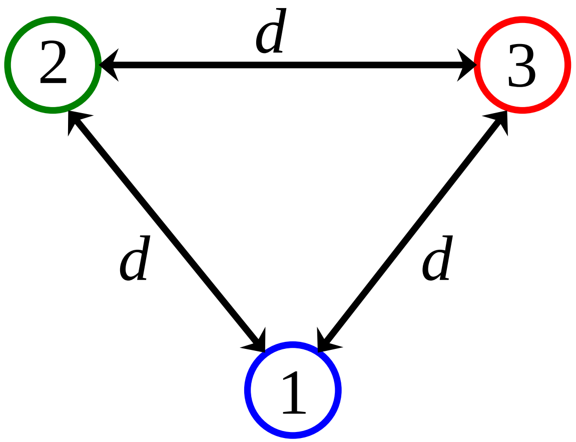
Example 4

Consider the situation shown below. The coefficient of kinetic friction between the blocks and the surface is 0.3.
Calculate the acceleration of the blocks.



Example 5

Shown in the figure are three planets. Determine the resultant gravitational force on planet 1 due to the presence of the other two.



Mass of planet 1 = 6.0×10^{20} kg

Mass of planet 2 = 2.6×10^{22} kg

Mass of planet 3 = 4.5×10^{21} kg

$d = 2.0 \times 10^{12}$ km

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

FIGURING PHYSICS

She holds a book stationary against the wall as shown. Friction on the book by the wall acts

(A) upward
(B) downward
(C) can't say

Solving problems requiring the application of Newton's Second Law and the kinematic equations.

$$\vec{\mathbf{F}}_{\text{resultant}} = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_o + \vec{\mathbf{u}}t + \tfrac{1}{2}\vec{\mathbf{a}}t^2$$

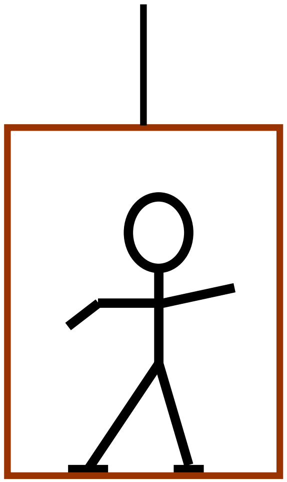
$$\vec{\mathbf{v}}(t) = \vec{\mathbf{u}} + \vec{\mathbf{a}}t$$

the link between the two is through the acceleration $\vec{\mathbf{a}}$

... but be careful to use a **consistent set of axes.**

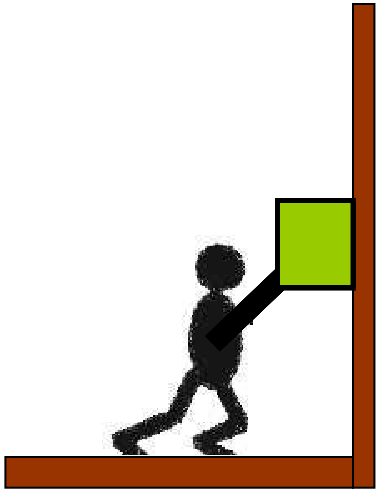
Example 6

The speed of a lift initially moving down at 10 m s^{-1} decreases to 4 m s^{-1} in a distance of 6.0 m.
Determine the force of the lift's floor on a 100 kg person standing in the lift.



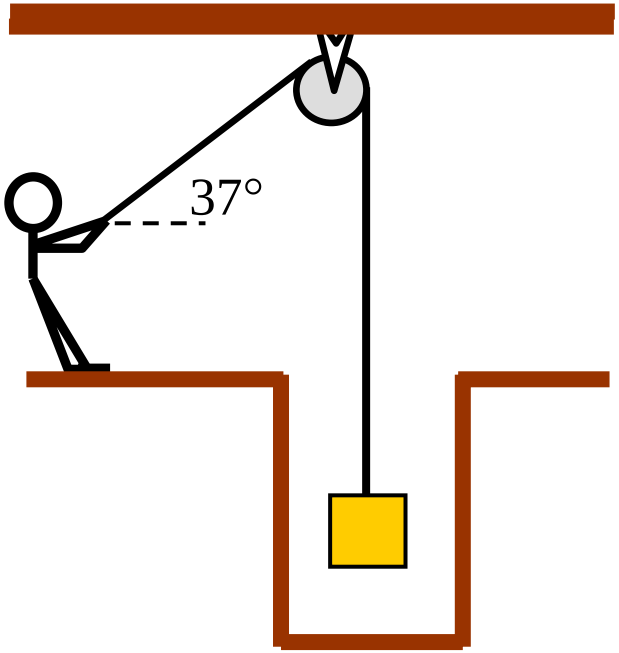
Example 7

The 20 kg crate shown is initially moving down at a speed of 2.0 m s^{-1}. Determine the magnitude of the average force exerted by the person in order to stop the crate's downward movement in a distance of 0.5 m. The person pushes in a direction 53° above the horizontal and the coefficient of kinetic friction between the crate and the wall is 0.40 .



Example 8

A farmer wishes to lift a heavy box by pulling a light rope that passes over a pulley and down to the box below. What is the minimum coefficient of static friction between her shoes and the floor that will prevent her from sliding across the surface while lifting the box at a constant speed? The farmer's mass is 70 kg and the box's weight is 250 N. Assume that there is no friction in the pulley.

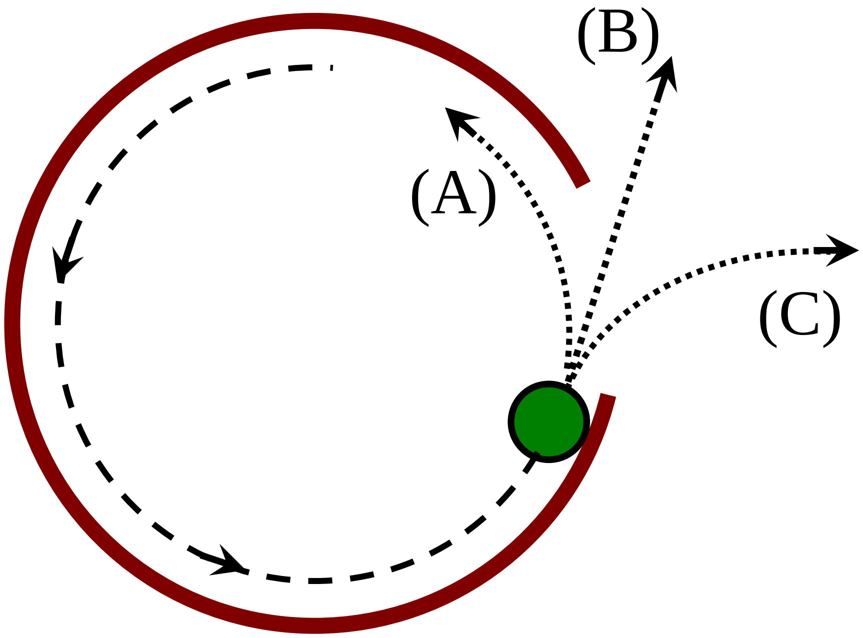


Example 9

A 40 kg box accidentally slides down a ramp and onto a level surface. On this surface, the box is initially moving at 9.0 m s^{-1} towards a large glass window 14 m away, when a man starts to push the box to prevent it from smashing the window. The force that the man exerts on the box is 100 N at an angle of 37° below the horizontal. (The man moves backward while exerting a force on the box.) A 40 N friction force opposes the box's motion. Will the box break the window?

Demonstration

A marble rolls along the inside of a horizontal circular loop. When the marble reaches the gap, which path will the marble follow?



<u>Rotational dynamics</u>

Previously we saw that when we observe a mass rotating in a circle, then there must be a centripetal acceleration and hence a **centripetal force** acting towards the centre of the circle.

$$\vec{\mathbf{a}}_c = \frac{v^2}{r}\hat{\mathbf{r}} \qquad \text{and} \qquad \vec{\mathbf{F}}_c = m\frac{v^2}{r}\hat{\mathbf{r}}$$

In each situation, some force or forces must have a component towards the centre of the circle and hence "provide" the centripetal force. In other words, the centripetal force is <u>not</u> an "extra force" ... it is the name given to the resultant force in the direction of the centre of the circle.

Newton's Second Law for rotational dynamics:

$$\vec{\mathbf{F}}_{\text{resultant}} = \sum m\vec{\mathbf{a}}$$

$$F_c\hat{\mathbf{r}} = \sum ma\,\hat{\mathbf{r}}$$

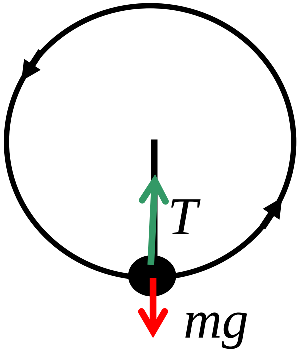
$$\boxed{m\frac{v^2}{r}\hat{\mathbf{r}} = \sum ma\,\hat{\mathbf{r}}}$$

Some examples:

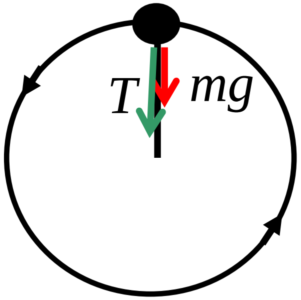
(a) bottom of circle

(b) top of circle

Ball moving at constant speed in a vertical circle of radius r:

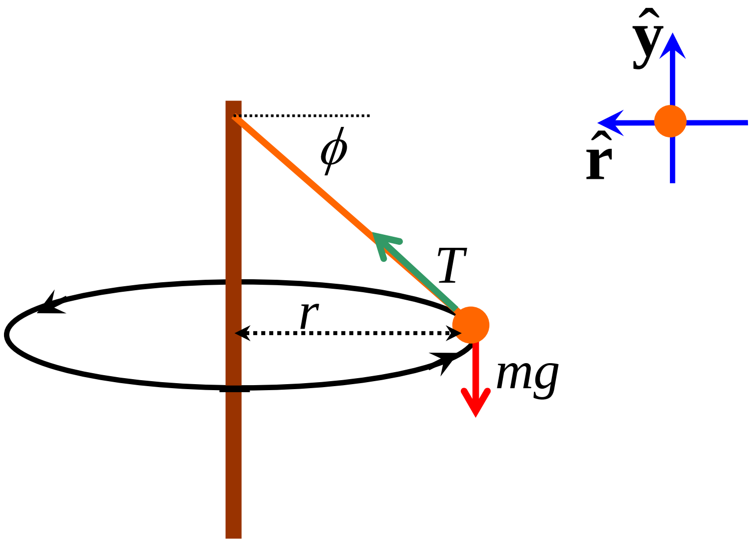


$$m\frac{v^2}{r} = T - mg$$



$$m\frac{v^2}{r} = T + mg$$

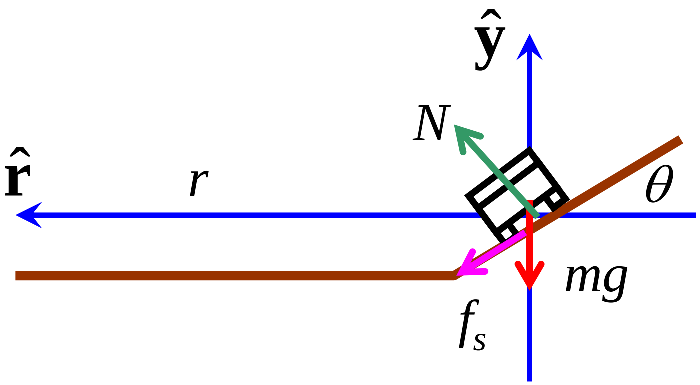
(c) A mass at the end of a rope moves in a circle around a pole at constant speed *v*.



$$\hat{\mathbf{r}} \; : \quad m\frac{v^2}{r} = T\cos\phi$$

$$\hat{\mathbf{y}} \; : \quad 0 = T\sin\phi - mg$$

(d) View from the back of a car traveling at constant speed *v* on a banked highway



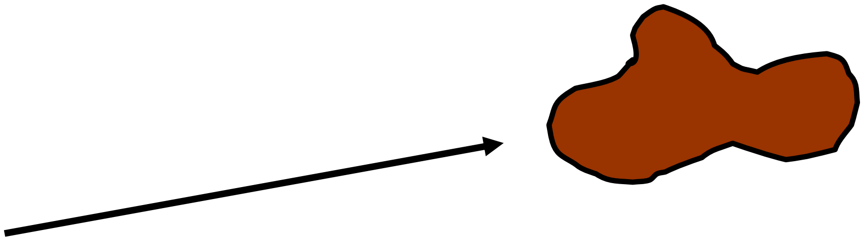
$$\hat{\mathbf{r}} \; : \quad m\frac{v^2}{r} = N\sin\theta + f_s\cos\theta$$

$$\hat{\mathbf{y}} \; : \; 0 = N\cos\theta - f_s\sin\theta - mg$$

Example

A car is traveling around a circular corner on a horizontal road. If the coefficient of static friction between the tires of the car and the road is 0.60, at what maximum speed can a 1000 kg car round a level 50 m radius curve without slipping?

<u>Extended bodies and centre of mass</u>



(can vibrate, rotate, etc.)

An extended body has size ... so far we have only considered the motion of a single particle •, but an extended body is made up of many particles.

We can show that if a body rotates, or if several particles move relative to one another, there is <u>one</u> point that moves in the same path that a single particle would move if subjected to the same net force. This is the **centre of mass**.

... this point moves as if the all mass is concentrated at that point.

We define the centre of mass for a system of particles as:

$$\vec{\mathbf{r}}_{\mathrm{cm}} = \sum \frac{m_i \vec{\mathbf{r}}_i}{M}$$

total mass = $\sum m_i$

$$x_{cm} = \sum \frac{m_i x_i}{M} \quad ; \quad y_{cm} = \sum \frac{m_i y_i}{M} \quad ; \quad z_{cm} = \sum \frac{m_i z_i}{M}$$

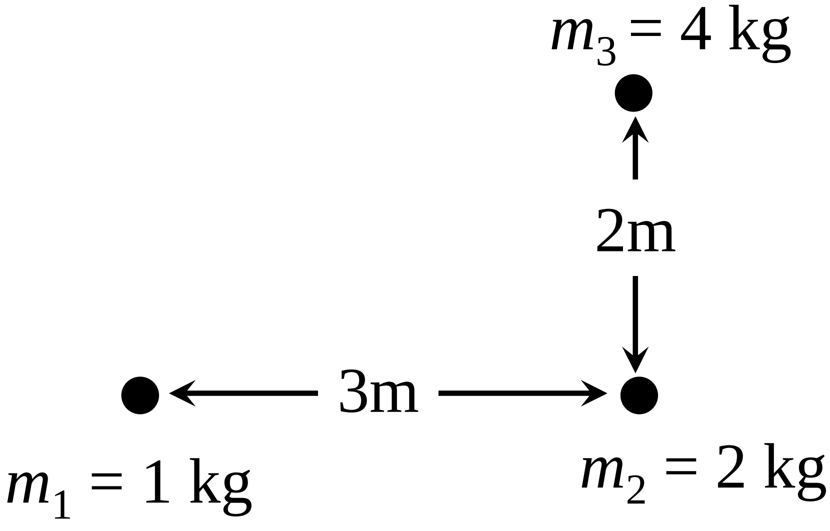
then $\vec{\mathbf{r}}_{\mathrm{cm}} = x_{\mathrm{cm}} \hat{\mathbf{i}} + y_{\mathrm{cm}} \hat{\mathbf{j}} + z_{\mathrm{cm}} \hat{\mathbf{k}}$

For an extended body (a continuous mass distribution) :

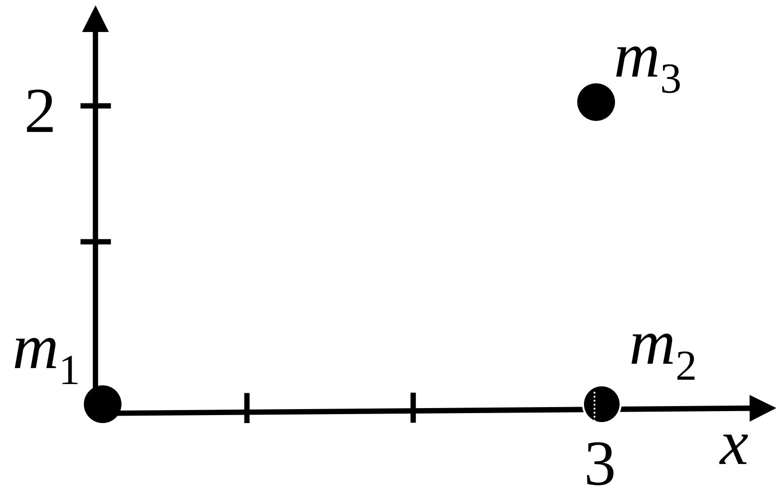
$$\vec{\mathbf{r}}_{\mathrm{cm}} = \frac{1}{M} \int \vec{\mathbf{r}} dm \quad = \quad \frac{1}{M} \int x dm \; \hat{\mathbf{i}} \; + \; \frac{1}{M} \int y dm \; \hat{\mathbf{j}} \; + \; \frac{1}{M} \int z dm \; \hat{\mathbf{k}}$$

Example

Find the centre of mass of the distribution of particles shown:



Choose a coordinate axes (<u>any</u> will do)

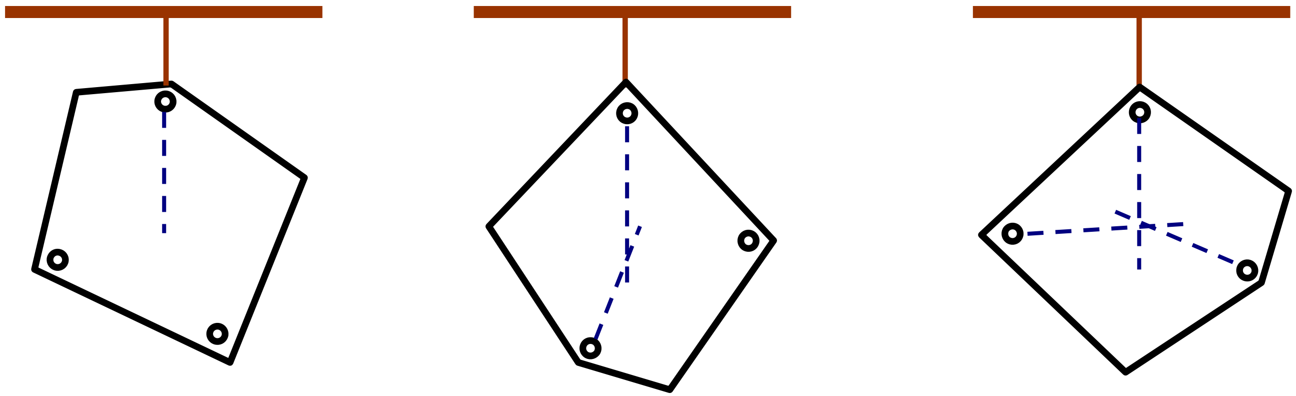


$$x_{\mathrm{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(1)(0) + (2)(3) + (4)(3)}{1 + 2 + 4} = \frac{18}{7} = 2.57$$

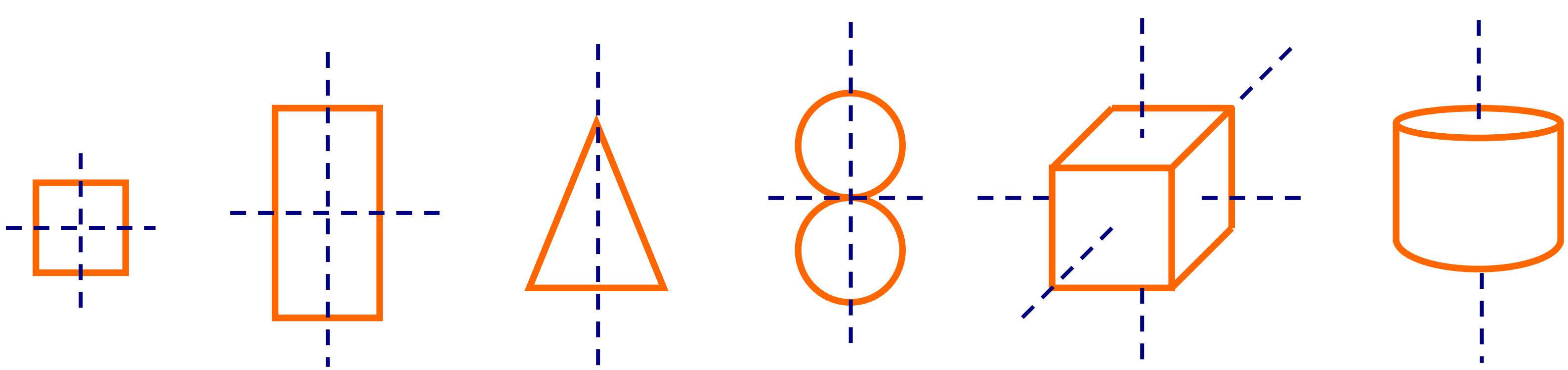
$$y_{\mathrm{cm}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(1)(0) + (2)(0) + (4)(2)}{1 + 2 + 4} = \frac{8}{7} = 1.14$$

$\therefore \quad \vec{\mathbf{r}}_{\mathrm{cm}} = (x,y) = (2.57, 1.14)$ m <u>in this coordinate system.</u>

A body free to rotate about a support will hang so that its centre of mass is vertically below support.



The centre of mass of a symmetrical object must lie on the line of symmetry.



Motion of the centre of mass

The importance of centre of mass lies in the fact that the motion of the centre of mass for a system of particles (or extended body) can often be described simply since it is related to net force on the system.

Consider n particles of total mass M which remains constant.

Then: $$M\vec{\mathbf{r}}_{\mathrm{cm}} = \sum m_i \vec{\mathbf{r}}_i$$

$$\therefore M\frac{d\vec{\mathbf{r}}_{\mathrm{cm}}}{dt} = \sum m_i \frac{d\vec{\mathbf{r}}_{\mathrm{i}}}{dt}$$

or $$M\vec{\mathbf{v}}_{\mathrm{cm}} = \sum m_i \vec{\mathbf{v}}_i$$

Velocity of i^{th} particle of mass m

Velocity of centre of mass

Motion of the centre of mass continued

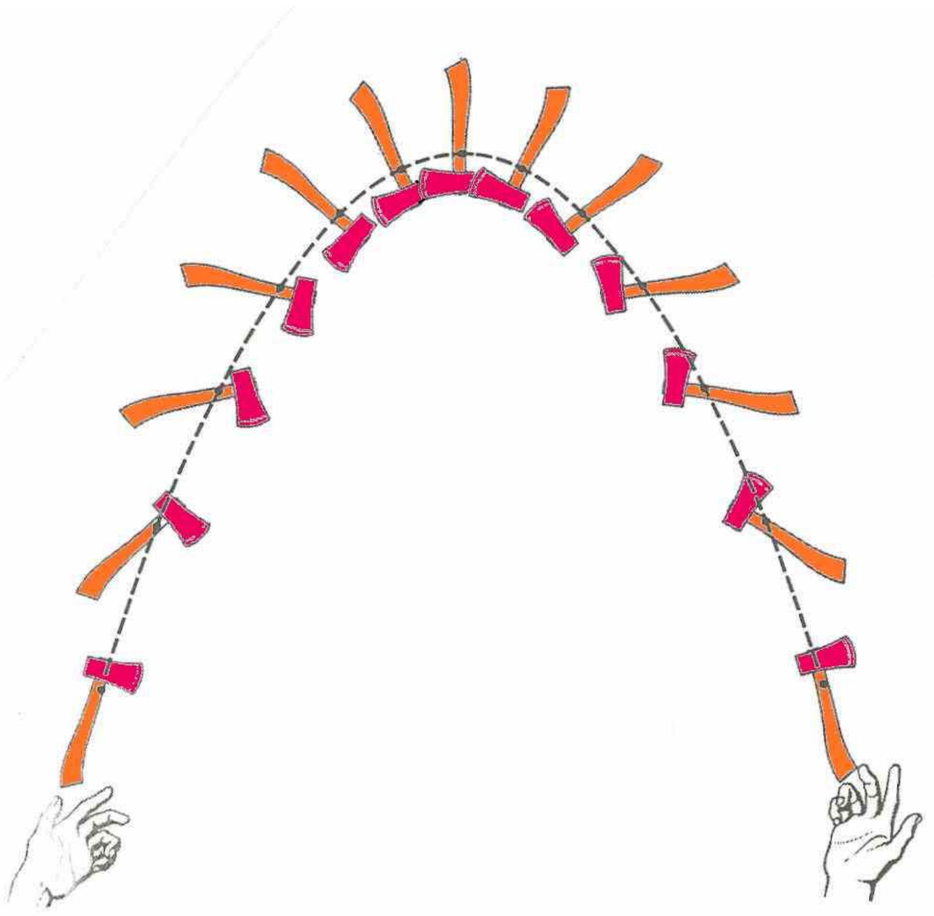
Then: $$M\frac{d\vec{\mathbf{v}}_{\mathrm{cm}}}{dt} = \sum m_i \frac{d\vec{\mathbf{v}}_{\mathrm{i}}}{dt}$$

$$M\vec{\mathbf{a}} = \sum \mathrm{m_i}\vec{\mathbf{a}}_i$$

Thus: $$M\vec{\mathbf{a}} = \sum \vec{\mathbf{F}}_i$$

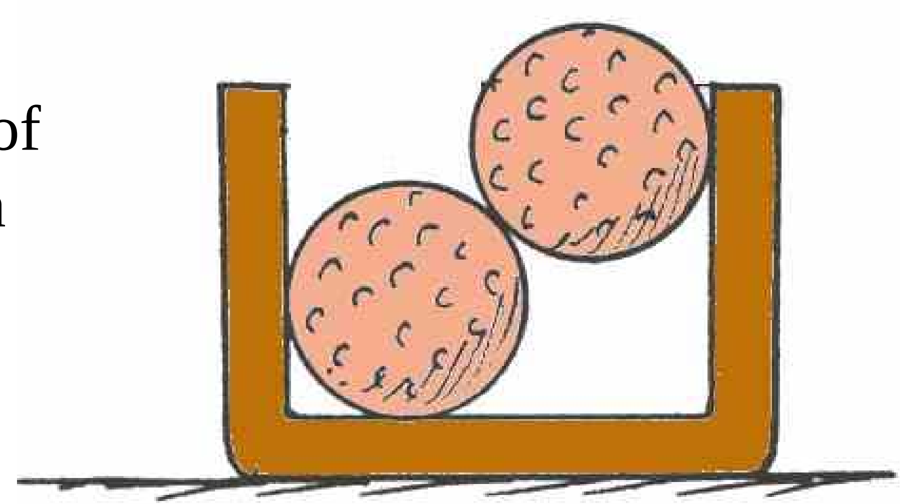
Therefore the vector sum of all the forces acting on the system equals the total mass of the system times the acceleration of the centre of mass.

The centre of mass moves as though all the mass were concentrated there, and the external forces are all applied at the centre of mass. All the internal forces cancel in action-reaction pairs.



Figuring Physics

Consider two golf balls in a massive cylindrical cup as shown. The arrangement is stable. If the mass of the cup were to suddenly approach zero, then the cup of balls would



(A) tip clockwise
(B) tip anticlockwise
(C) remain in stable equilibrium