PHY1023H

Tools and Skills Mechanics Rectilinear motion

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… covering pages 92 to 101 in *Reese*

Do the following problems:
3.1 ; 3.5 ; 3.9 ; 3.13 ; 3.17 ;
3.21 ; 3.25 ; 3.29 ; 3.33 ; 3.37 ;
3.41 ; 3.45 ; 3.49 ; 3.53 ; 3.57 ;
3.61 ; 3.65 ; 3.69

… solutions on vula …

Mechanics

... has to do with the way bodies interact with each other.

Coming up in this section ...

- Kinematics in one dimension (including falling bodies)
- Kinematics in two dimensions (projectiles)
- Relative motion
- Rotational kinematics
- Forces and Newton's Laws (linear dynamics)
- Rotational dynamics
- Centre of mass
- Work and Energy
- Linear momentum
- Torque, moments and angular momentum
- Statics

Modeling tools for mechanics:

Words
Pictures
Physics diagrams
Mathematics

Not the best place to start →

Very often in physics we need to model the motion of moving bodies. In this course we will in most cases consider the body to be a point particle. We will discuss this later. To start we need to think about how to visually represent the motion of a particle in different ways, before we try to model this motion mathematically. Visual representations can be very useful models since they allow us to make sense of things conceptually.

Consider a box sliding on a frictionless surface. If you were asked to **describe** the motion of the box, there are a number of ways that you could use … these include …
… words … pictures … mathematics …

However, in order to understand a particular situation, this very often requires us to **represent** the situation in a number of different ways … and each of these representations carry very particular types of information about the system in question.

Newtonian mechanics

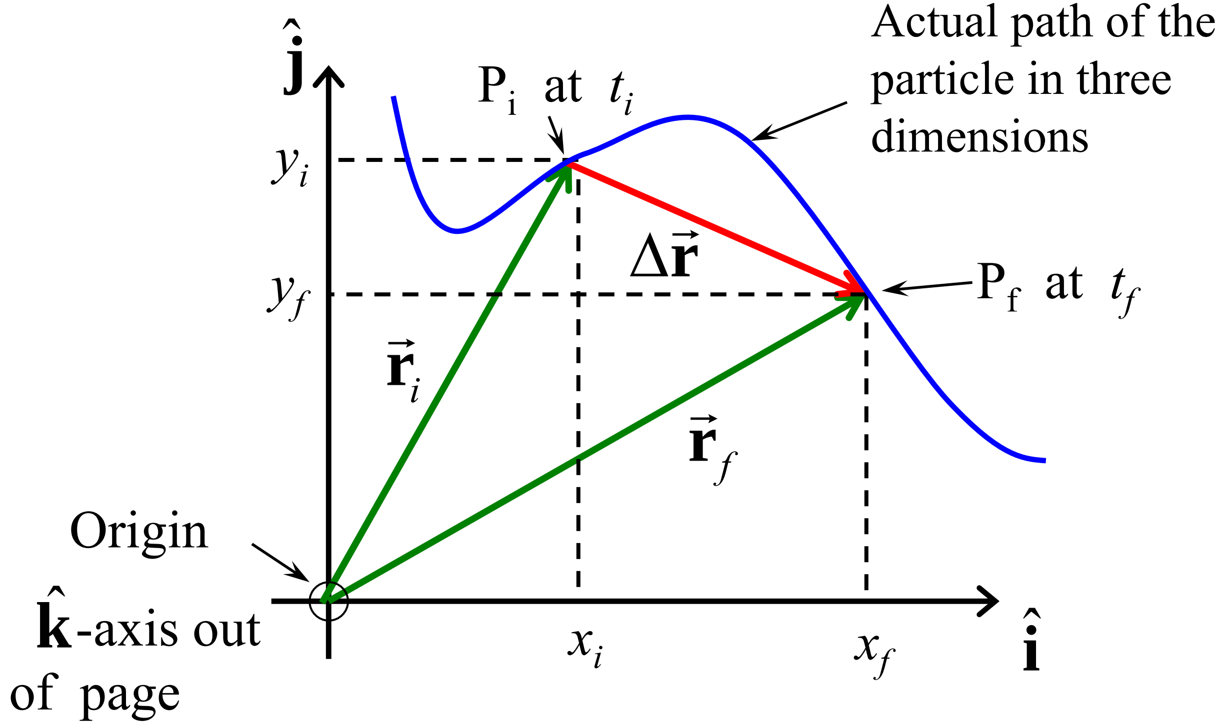
The "flavour" of mechanics that we will use is called "Newtonian mechanics" … developed by Sir Isaac Newton …

… which has a number of conditions which are important …

… Nature is assumed to be **continuous** (which was shown not to be quite true by Einstein's Special Relativity, and quantum mechanics.)

… the equations of motion are applicable only to **point particles** (in most of the situations we will consider in this course).

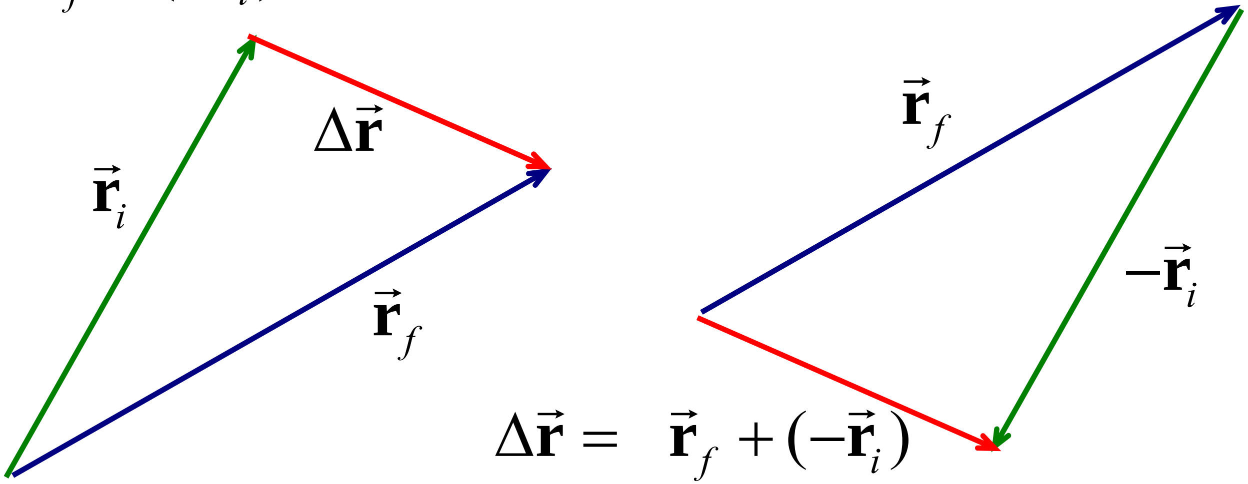
Newtonian kinematics



Consider a particle that follows the curved (blue) path in space.
At time t_i it is at position $\mathrm{P_i}$ and time t_f it is at position $\mathrm{P_f}$.
To describe the motion of the particle, we use a three dimensional Cartesian coordinate system as shown.
Therefore at $t = t_i$, the particle is at position $\vec{\mathbf{r}}_i = x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}$
and $t = t_f$, the particle is at position $\vec{\mathbf{r}}_f = x_f\hat{\mathbf{i}} + y_f\hat{\mathbf{j}} + z_f\hat{\mathbf{k}}$

The **displacement vector** is

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i = \left(x_f - x_i\right)\hat{\mathbf{i}} + \left(y_f - y_i\right)\hat{\mathbf{j}} + \left(z_f - z_i\right)\hat{\mathbf{k}}$$
$$= \vec{\mathbf{r}}_f + (-\vec{\mathbf{r}}_i)$$



Of course $\left|\Delta\vec{\mathbf{r}}\right|$ is <u>not</u> necessarily the distance from P_i to P_f.

In general an **instantaneous position vector** $\vec{\mathbf{r}}(t)$ describes the position of a particle at a particular instant in time relative to the origin of a set of coordinate axes:

$$\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$$

The **average velocity** vector $\vec{\mathbf{v}}_{av} = \frac{\Delta \vec{\mathbf{r}}}{\Delta t} = \frac{\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i}{t_f - t_i}$

The **instantaneous velocity** vector $\vec{\mathbf{v}}(t) = \frac{d\vec{\mathbf{r}}(t)}{dt}$

$$= \frac{dx(t)}{dt}\hat{\mathbf{i}} + \frac{dy(t)}{dt}\hat{\mathbf{j}} + \frac{dz(t)}{dt}\hat{\mathbf{k}}$$

The **average acceleration** vector $\vec{\mathbf{a}}_{av} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i}$

The **instantaneous acceleration** vector $\vec{\mathbf{a}}(t) = \frac{d\vec{\mathbf{v}}(t)}{dt}$

$$= \frac{d^2x(t)}{dt^2}\hat{\mathbf{i}} + \frac{d^2y(t)}{dt^2}\hat{\mathbf{j}} + \frac{d^2z(t)}{dt^2}\hat{\mathbf{k}}$$

Kinematics with constant acceleration

We introduce of two kinematics equations of motion for the case of constant acceleration:

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \tfrac{1}{2}\vec{\mathbf{a}}t^2$$

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$\vec{\mathbf{r}}(t)$: position vector at time t

$\vec{\mathbf{r}}_0 = \vec{\mathbf{r}}(t=0)$: position vector at time $t = 0$ (the initial position)

$\vec{\mathbf{v}}(t)$: velocity vector at time t

$\vec{\mathbf{v}}_0 = \vec{\mathbf{v}}(t=0)$: velocity vector at time $t = 0$ (the initial velocity)

$\vec{\mathbf{a}}$: acceleration vector (constant)

The displacement vector is then $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}(t) - \vec{\mathbf{r}}_0$

When the motion is only in one dimension, we can use slightly different notation:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_0 + \vec{\mathbf{v}}_{x0}t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2 \qquad \vec{\mathbf{v}}_x(t) = \vec{\mathbf{v}}_{x0} + \vec{\mathbf{a}}_x t$$

$$\vec{\mathbf{y}}(t) = \vec{\mathbf{y}}_0 + \vec{\mathbf{v}}_{y0}t + \tfrac{1}{2}\vec{\mathbf{a}}_y t^2 \qquad \vec{\mathbf{v}}_y(t) = \vec{\mathbf{v}}_{y0} + \vec{\mathbf{a}}_y t$$

…or in scalar form:

$$x(t) = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 \qquad v_x(t) = v_{x0} + a_x t$$

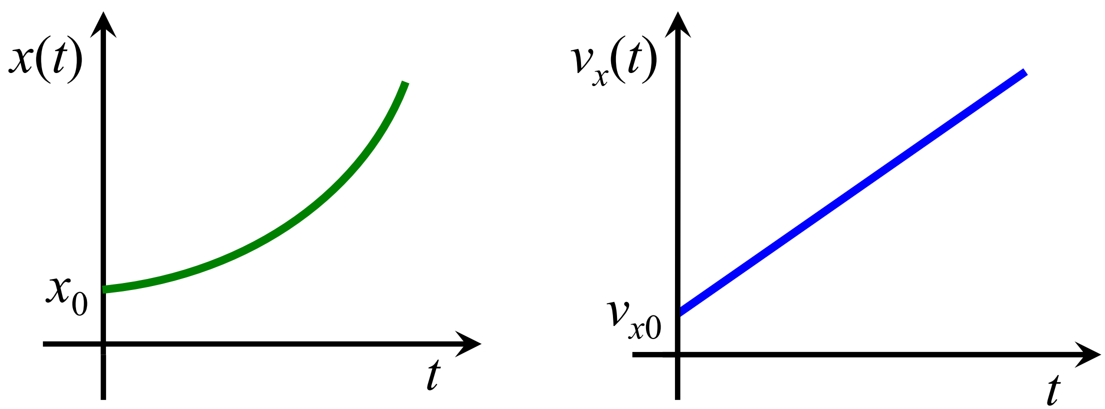
$$y(t) = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \qquad v_y(t) = v_{y0} + a_y t$$

Nature is continuous

The equations of motion arising from Newtonian mechanics

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_0 + \vec{\mathbf{v}}_{x0}t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2 \qquad \vec{\mathbf{v}}_x(t) = \vec{\mathbf{v}}_{x0} + \vec{\mathbf{a}}_x t$$

… allow one to calculate the position and velocity of a particle at any instant in time … they are complete descriptions of the **continuous motion** of the particle …



If one knows the initial conditions, then one is able to predict the motion at some time in the future … the equations are **deterministic** … later on, quantum mechanics showed that nature should be understood as being probabilistic and the determinism of Newton's mechanics applies only to the macro scale.

Point particles

If you look in your textbook, and most other first year physics textbooks, they are filled with images are cars, boats, planes, …

… the authors and publishers do this in order to add interest and colour and show applicability to the real world!

However, when you see this (and sometimes we will draw “real” objects as well)…

… what you should do is collapse it down to this …

Later on in this course, the **shape** of the object does matter in certain cases such as when we consider the rotation of rigid bodies, but for now … point particles only!

Other representations of motion

In most cases, when faced with a particular situation to analyze, it is often useful to think about the situation in other ways first, before diving into using (abstract) equations …
… there are a number of **diagrammatic representations** which are often helpful to understand what is going on and to visualize the motion.
It is not sufficient to be able to only calculate the correct answer … you need to also understand the situation fully … which means being able to **describe** the motion in a variety of ways.

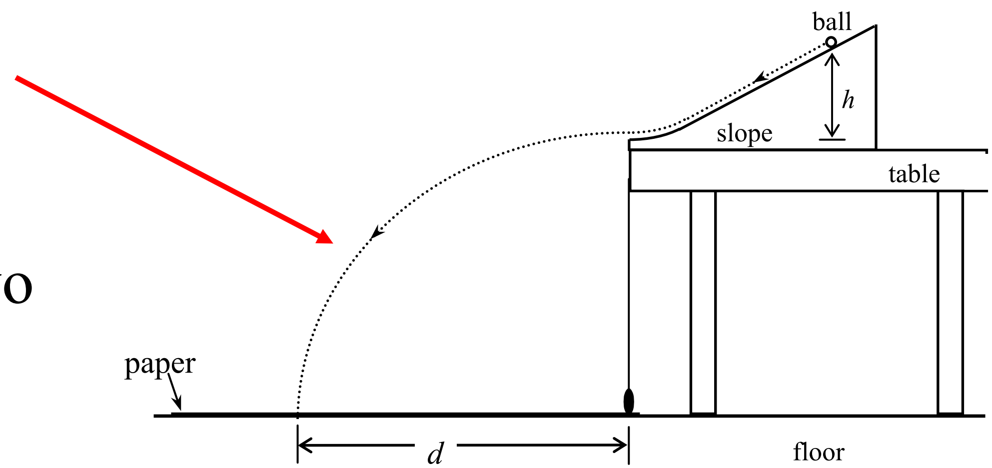
The difficulty in using visual-rich representations of a continuous motion is that we need to make choices with respect to what aspects of the motion we want to represent …

For example …

One way of representing the motion is to draw a continuous line, following every position of the object from its start to end positions … we can call this the "**pathline**" or simply the "path" of the object.

For the situation above:

Drawing the **pathline** can be very useful, especially when the object is moving in two dimensions, for example:



But the information that we can represent by a pathline is very limited.

We could also make a **movie** of the whole motion, but then we would need special equipment to store this information.

If we limit ourselves to representations that can be drawn on a piece of paper …

We could draw the object at its **initial position** and **final position**, and then add a whole lot of information onto that diagram …

Reese p. 98

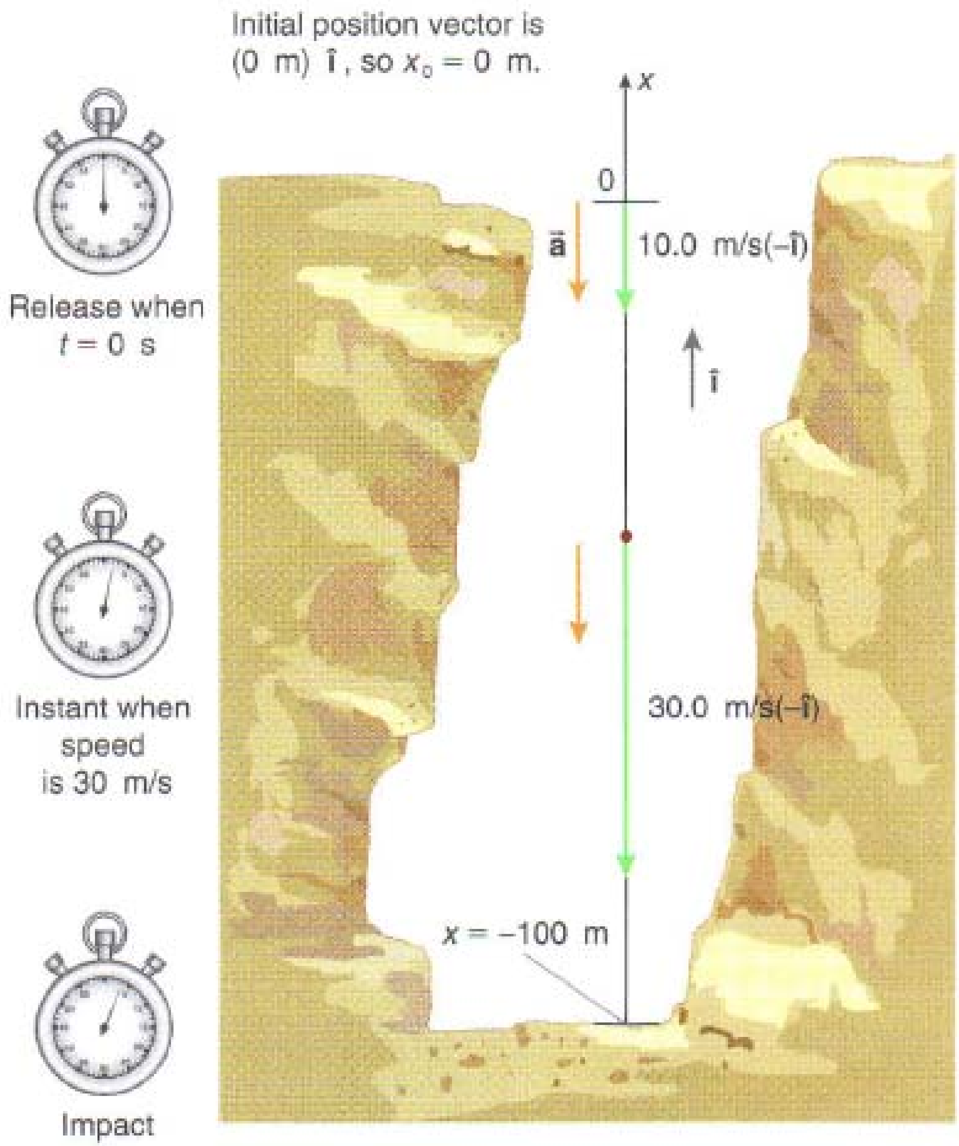


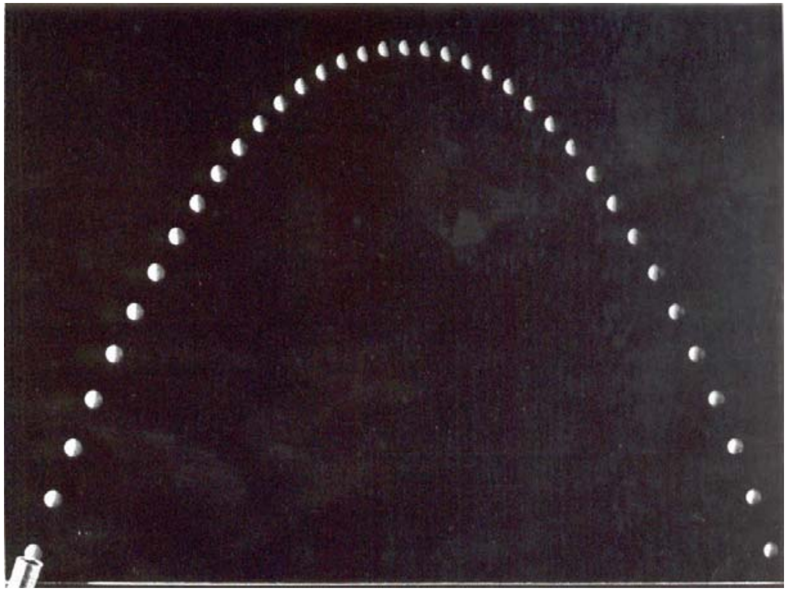
FIGURE 3.46

We will learn how to do this later …

Stroboscopic photographs

An intermediate step between a movie and only drawing the object at its initial and final positions, is to draw in the object at **many** positions along its path.

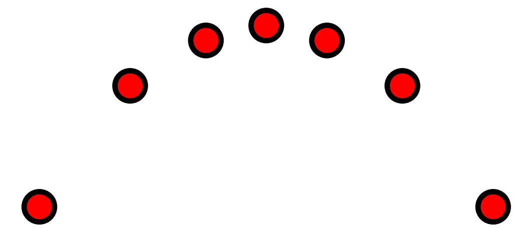
Look at the photograph alongside. It is called a **stroboscopic photograph** since a camera took a photograph ("snapshot") of the ball every Δt apart (synchronized with a strobe light flash) and then all the photographs are displayed on the same frame.



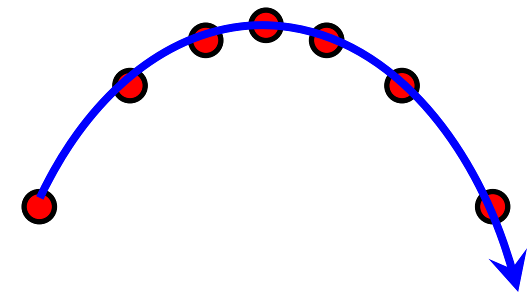
The time interval Δt could be a fraction of a second, of course.

Now we can make some observations …

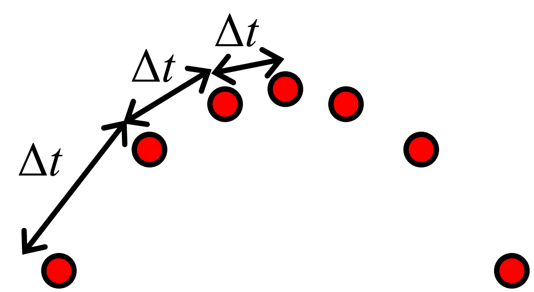
Taking a reduced case, for illustration:



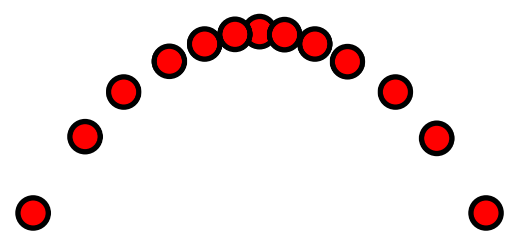
1. If we join up all the dots, we get the **pathline** (worldline) of the object.



2. The time between each photograph is Δt.

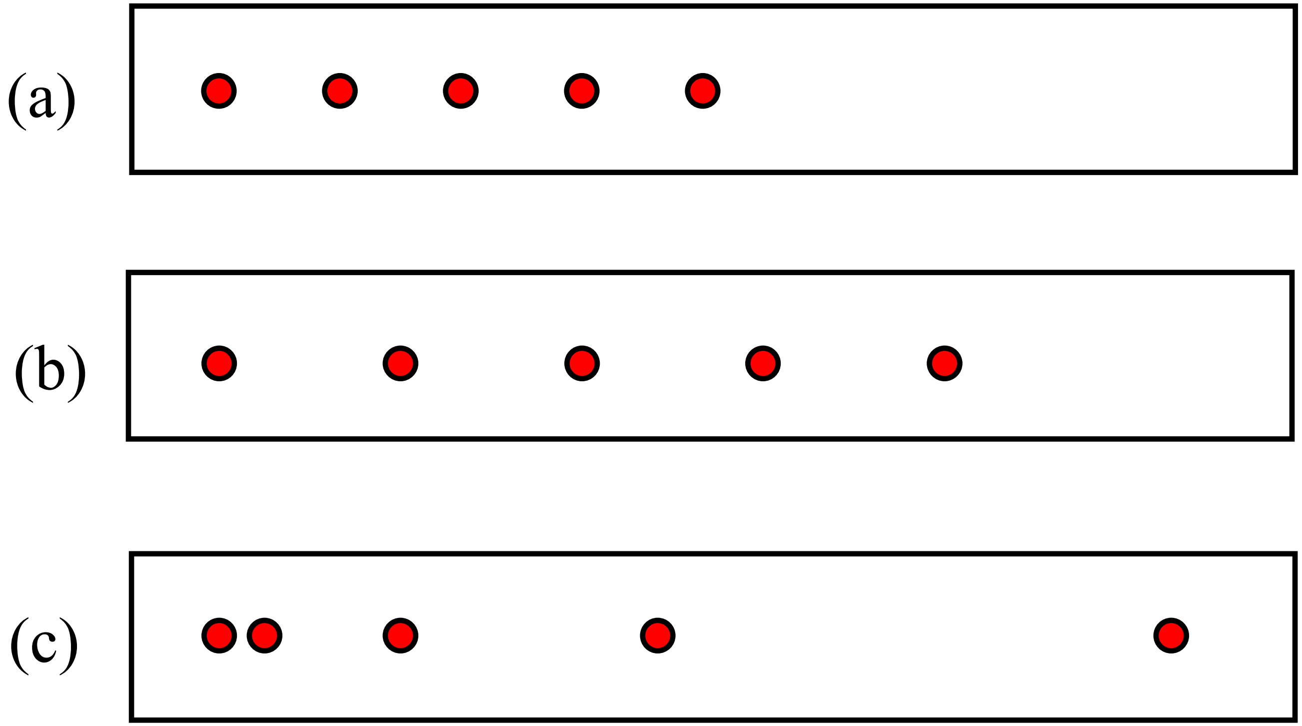


3. As we make Δt smaller and smaller, and hence take more photographs along the object’s path, then we approach a “continuous” movie … which of course is never truly continuous, it only appears that way to our eyes!



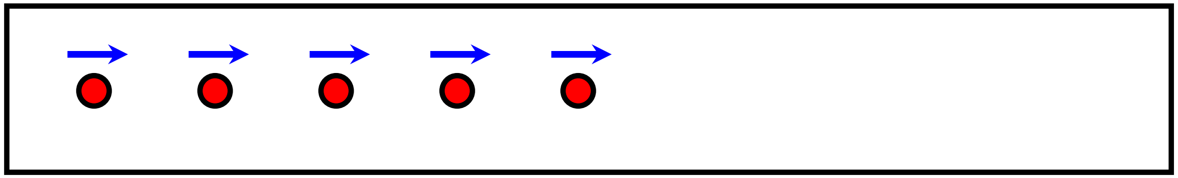
Now, here's the really useful thing about this representation …

Consider the three cases below where photographs of the ball were taken equal time periods apart and superimposed onto the same frame in each case. What can you say about the **acceleration** of the ball in each case?

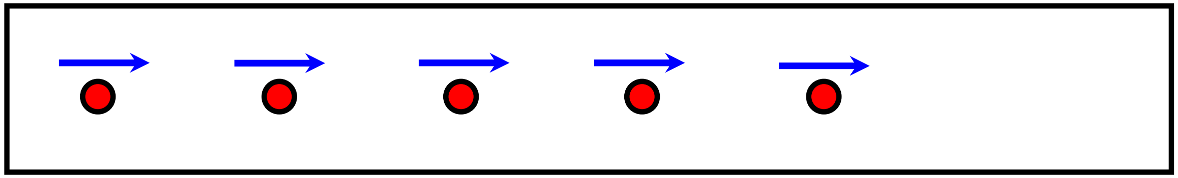


We will now draw in a vector next to each photograph to indicate the **magnitude and direction of the velocity** of the ball at that position … we would get something that looks like this:

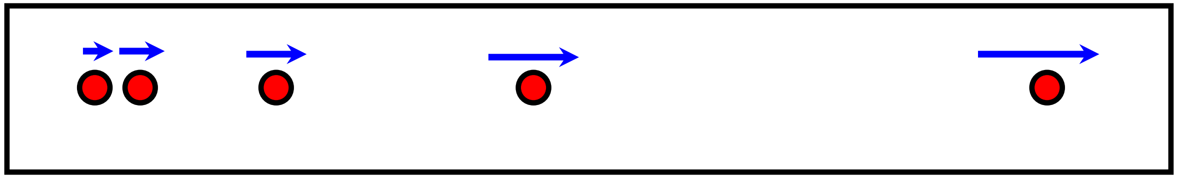
(a)



(b)



(c)



We are now able to represent quite a few attributes of the motion of the ball.

“Freeze frame” or “multiple snapshot” representation.

In the exercises that follow, and for the rest of this course, let’s agree that whenever we use this “stroboscopic photographic representation” we will always draw the object at **seven** positions, and always include the **initial and final positions**.

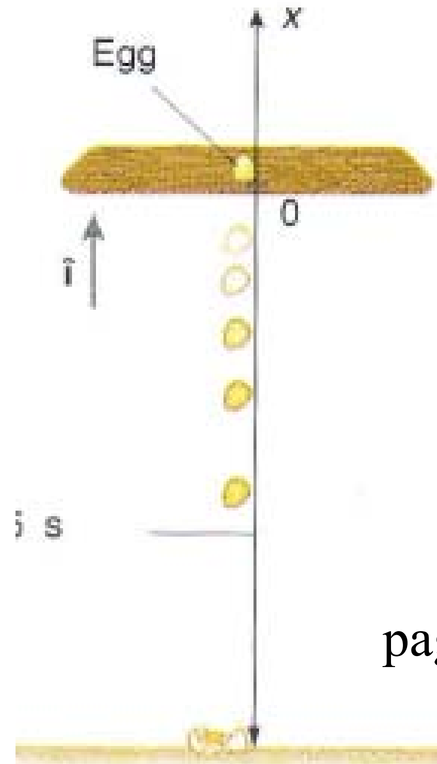
Furthermore, instead of writing, every time, “draw seven photographs, equally spaces in time, on the same frame, etc, etc. …”
… we will refer to this representation as the “**freeze frame**” or “**multiple snapshot**” representation.

Multiple snapshot representations in *Reese*

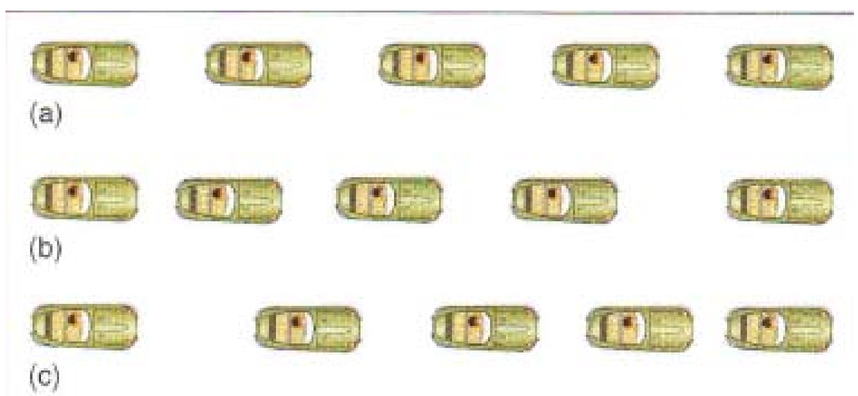
Reese uses this representation throughout Chapter 3:

FIGURE 3.33 Car slowing down in rectilinear motion.

page 91



page 95



page 105

Example 1:

You drop a ball from the top of a tall building at $t = t_{intial} = 0$ s and it reaches the ground at $t = t_{final}$.

(a) Draw the freeze frame representation of the motion of the motion from its initial to final positions.

(b) Draw in a vector next to each photograph to indicate the magnitude and direction of the velocity of the ball

(c) What can you say about the acceleration of the ball?

(d) Draw in the continuous path of the ball from its start to end position

(e) What can you say about the shape of the path?

Example 2:

A block of ice slides across a wet table at a constant speed v_0 m s^{-1}.
On the diagram below, the block is shown at times $t_i = 0$ and t_f.
(a) Draw the freeze frame representation of the motion of the block.
(b) Draw in a vector next to each position of the block to indicate the magnitude and direction of the velocity of the block.
(c) What can you say about the acceleration of the block?
(d) Draw in the continuous path of the block from its start to end position.
(e) What can you say about the shape of the path?

Example 3:

A block of ice slides down a wet slope from rest. On the diagram below, the block is shown at times $t_i = 0$ and t_f .

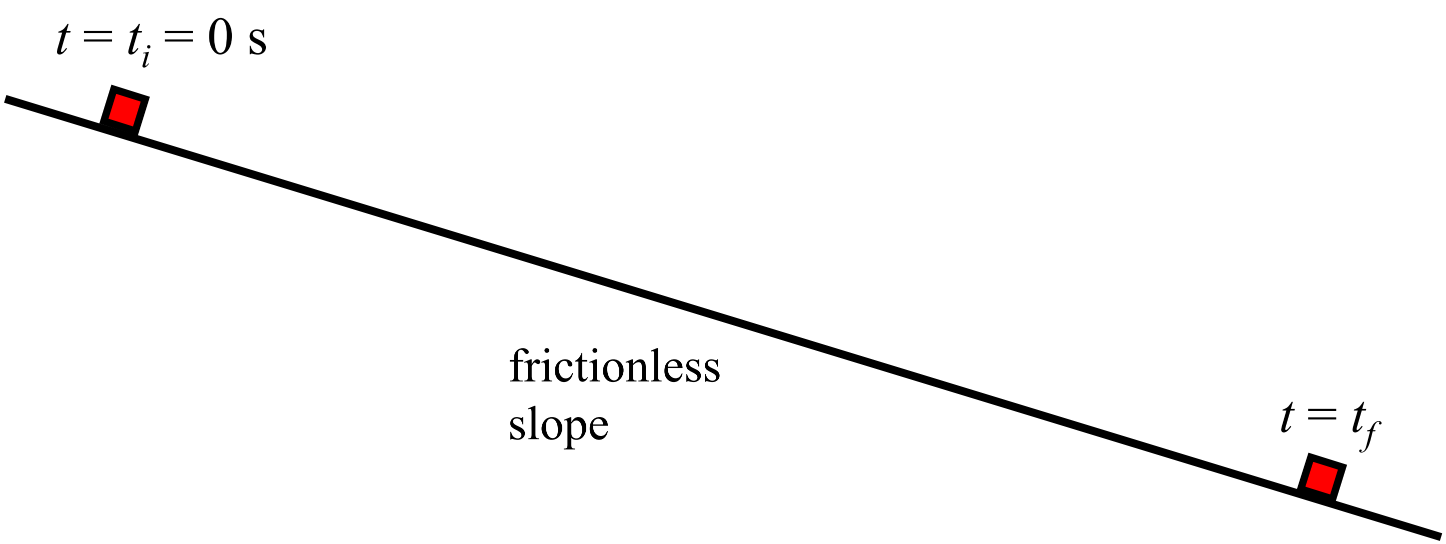
(a) Draw the freeze frame representation of the motion of the block. .

(b) Draw in a vector next to each position of the block to indicate the magnitude and direction of the velocity of the block.

(c) What can you say about the acceleration of the block?

(d) Draw in the continuous path of the block from its start to end position.

(e) What can you say about the shape of the path?



Example 4:

A wooden block slides across a table (with friction) at an initial speed v_0 m s^{-1} (at $t_i = 0$) and comes to rest at t_f. On the diagram below, the block is shown at times $t_i = 0$ and t_f .

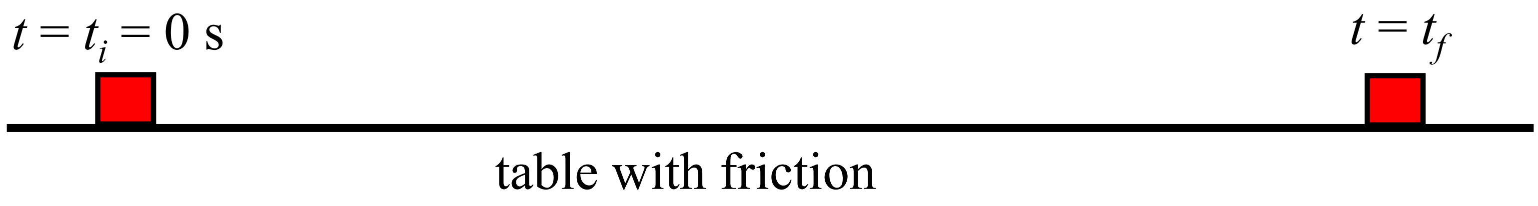
(a) Draw the freeze frame representation of the motion of the block. .

(b) Draw in a vector next to each position of the block to indicate the magnitude and direction of the velocity of the block.

(c) What can you say about the acceleration of the block?

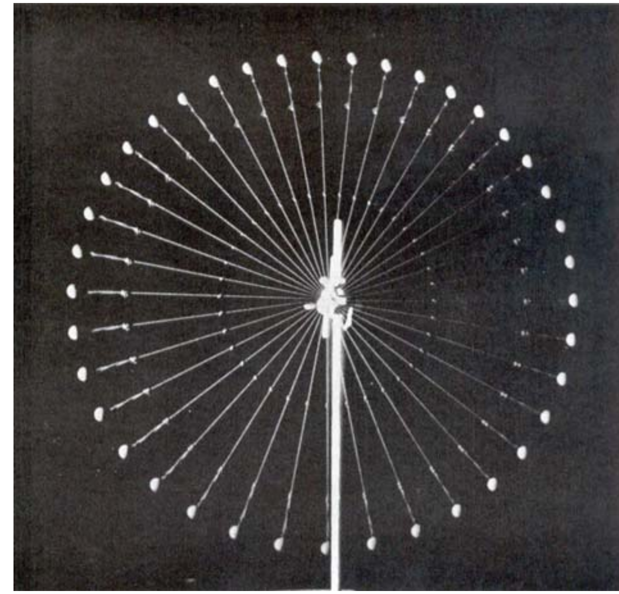
(d) Draw in the continuous path of the block from its start to end position.

(e) What can you say about the shape of the path?

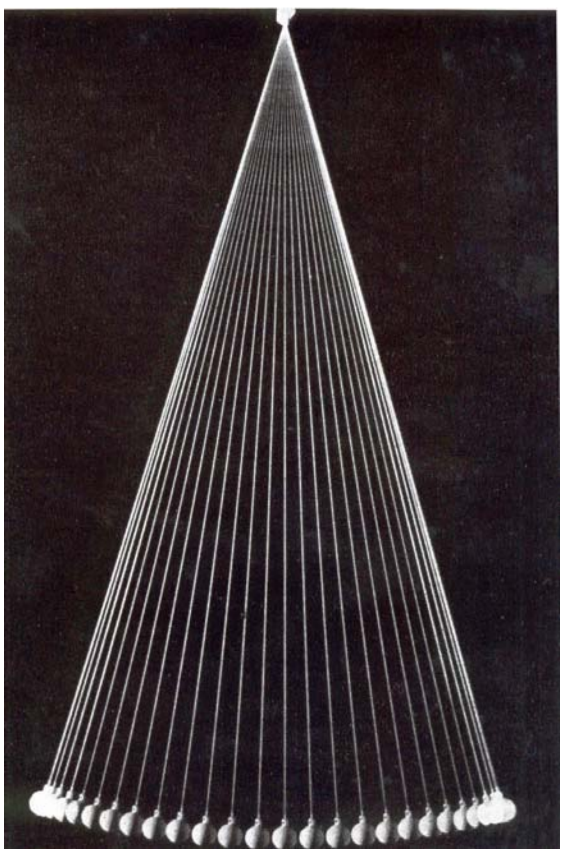


Here are three examples of motion in two dimensions. Explain what is going on in each case.

(b)



(a)



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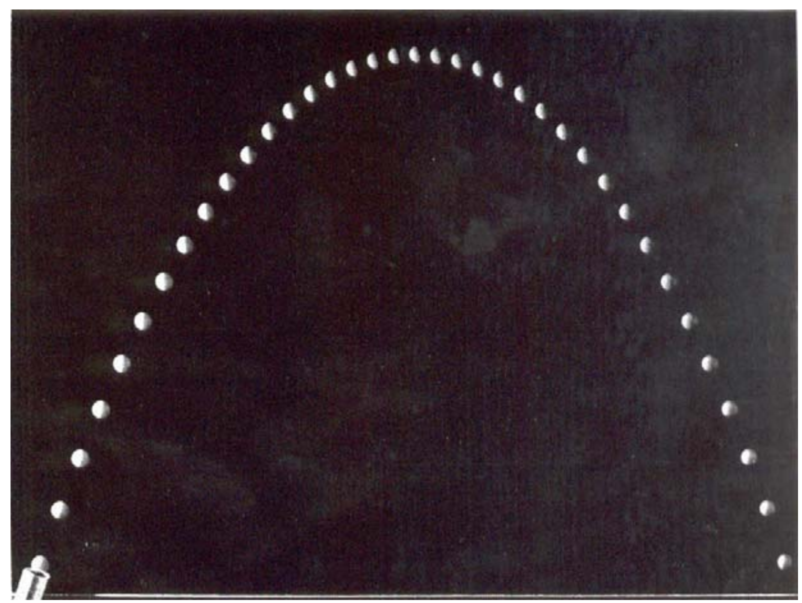
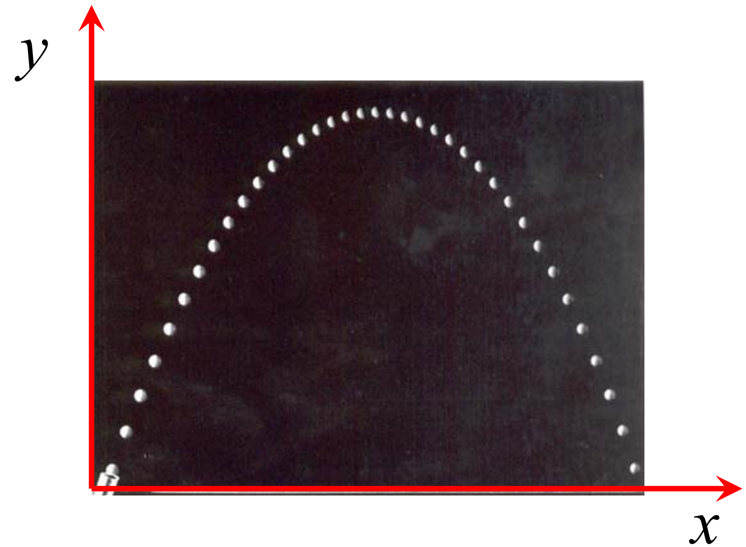


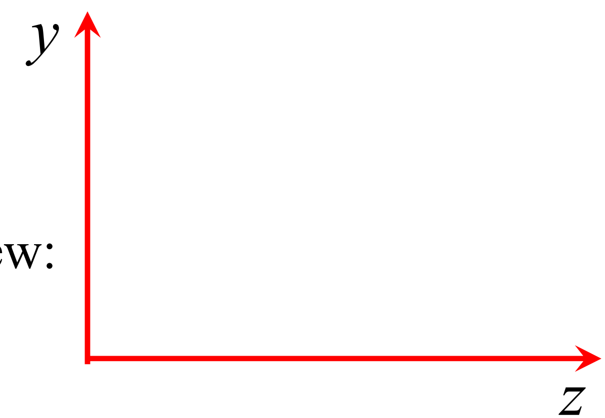
One needs to be careful with using the freeze frame representation since it is not always clear from which perspective we are viewing the motion …

For example, say that we are observing this projectile which is following a path on the *x*-*y* plane, with the *z*-axis into the page.

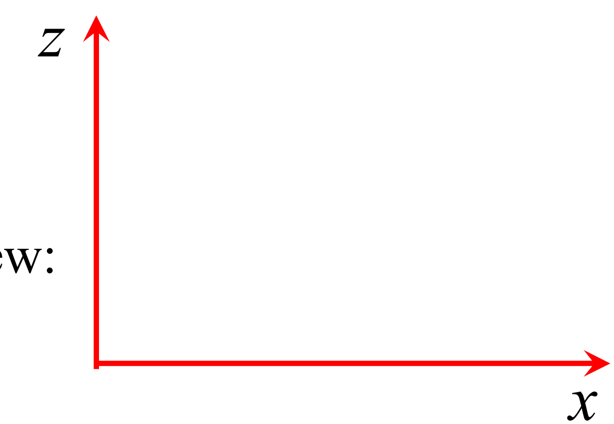


Draw what we would see below if we observed the same motion in the other two planes:

Side view:



Top view:

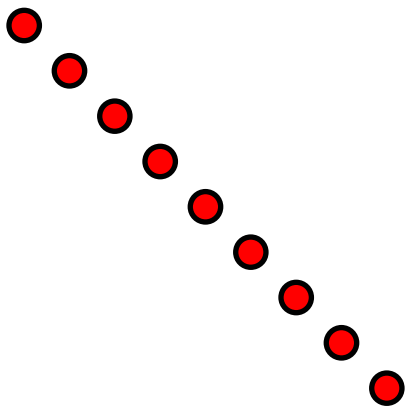


More freeze frame representations of motion …

Explain the motion in each case

(d)

(e)



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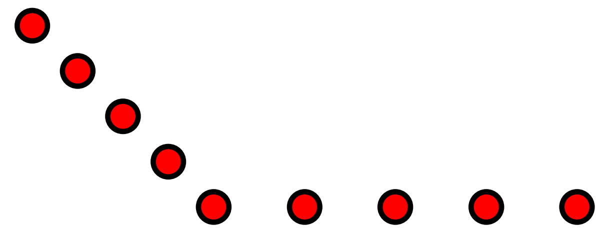
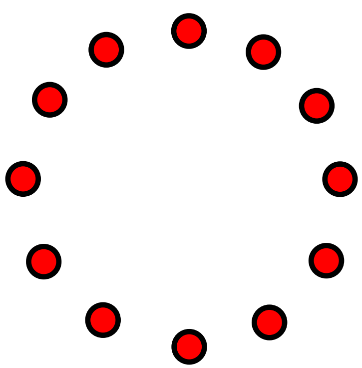


(g)



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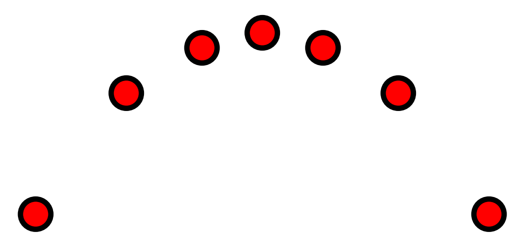


Using motion (velocity vector) diagrams

Say that in each case below, a velocity vector was drawn at different positions of a moving object. What can you say about the **acceleration** of the object in each case?

(a)

(b)

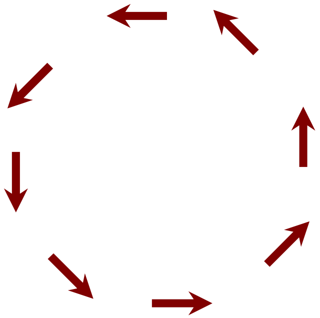
(c)

More velocity vector representations of motion…

Explain the motion in each case

(d)

(e)



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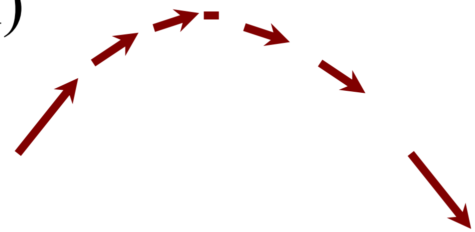
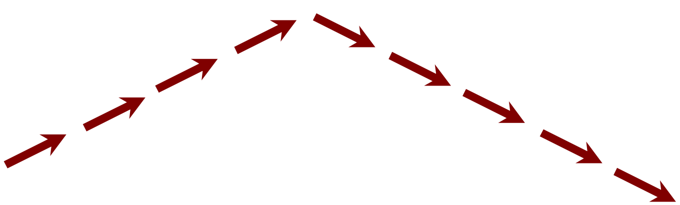


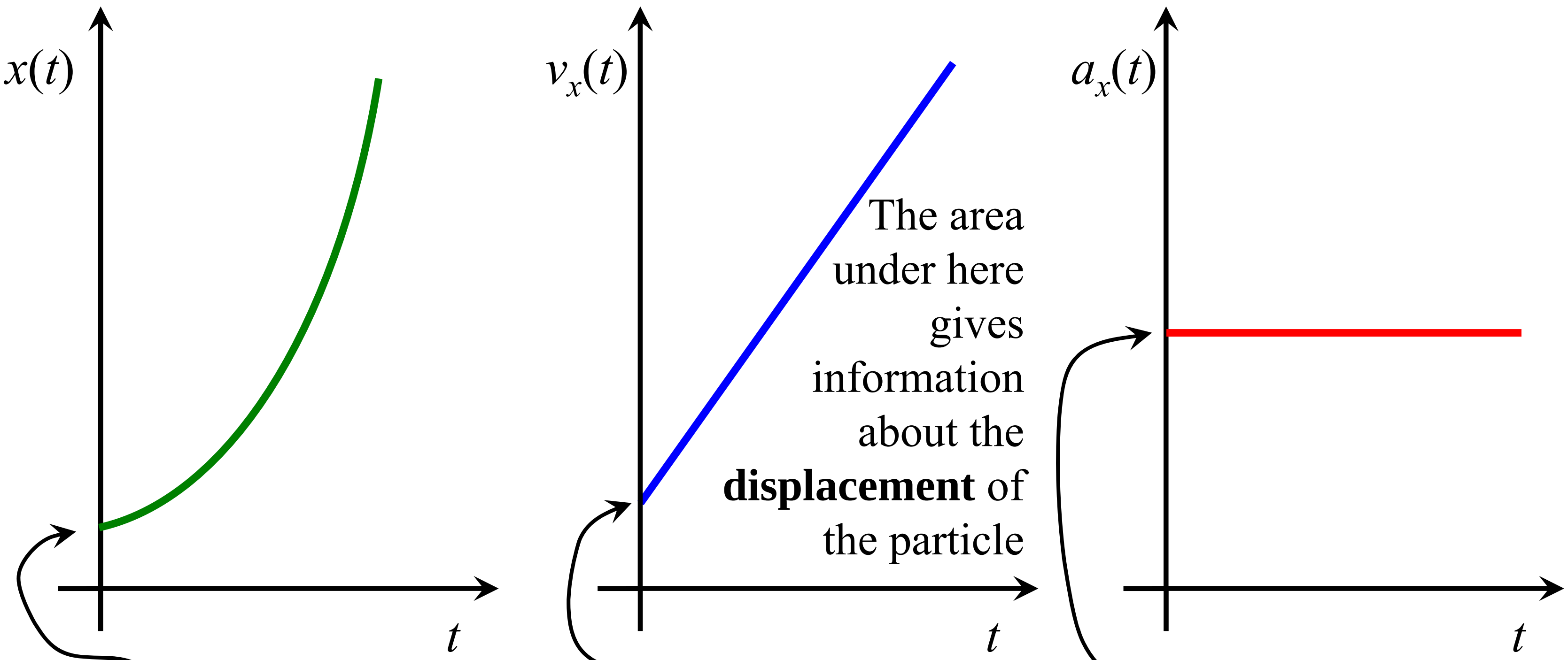
(g)

(h)



Graphical representations of motion

… for cases of constant (or zero) acceleration



$$x(t) = x_0 + v_{x0}t + \tfrac{1}{2}a_xt^2$$

$$\frac{d}{dt}x(t) = \frac{d}{dt}\left(x_0 + v_{x0}t + \tfrac{1}{2}a_xt^2\right) \rightarrow v_x(t) = v_{x0} + a_xt$$

$$\frac{d}{dt}v_x(t) = \frac{d}{dt}\left(v_{x0} + a_xt\right) \rightarrow a_x = \text{constant}$$

Using graphs of motion … Example 1

Consider the ***x*(*t*) versus time** graphs below, for an object moving in one dimension. In each different case:

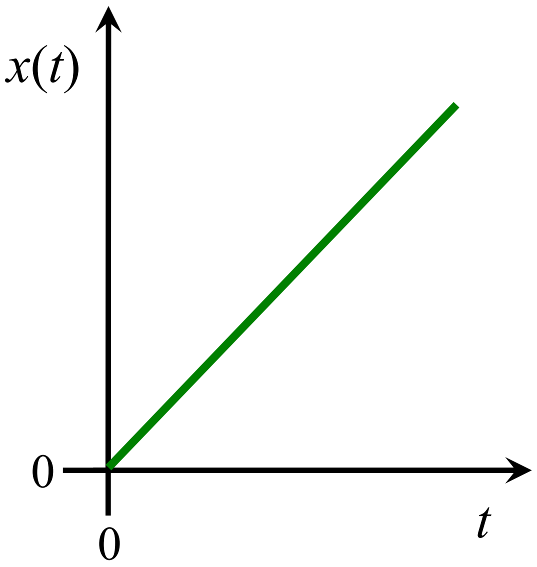
(i) write down the all the information that you can obtain from the graph to describe the motion of the object

(ii) provide an example of a situation where the motion of the object is represented by the graph

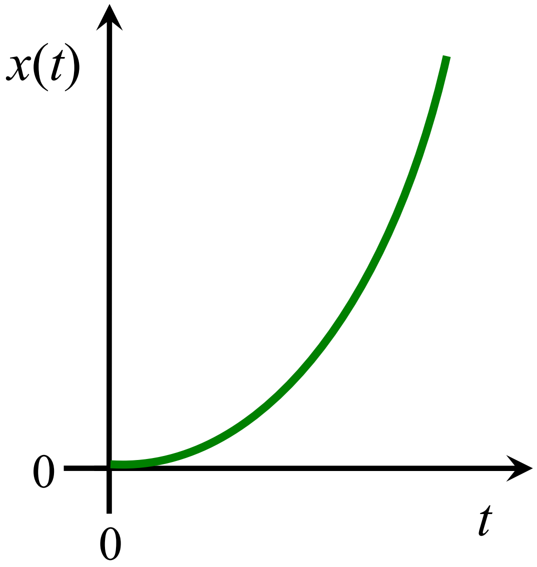
(iii) Draw the freeze frame photograph representation in each case (use a circle for the object)

(iv) use (iii) to sketch the corresponding **$v_x(t)$ vs t** and **$a_x(t)$ vs t** graphs.

(a)



(b)



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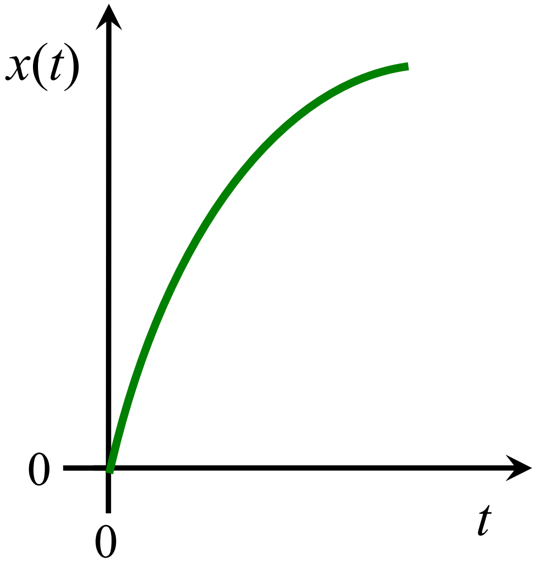

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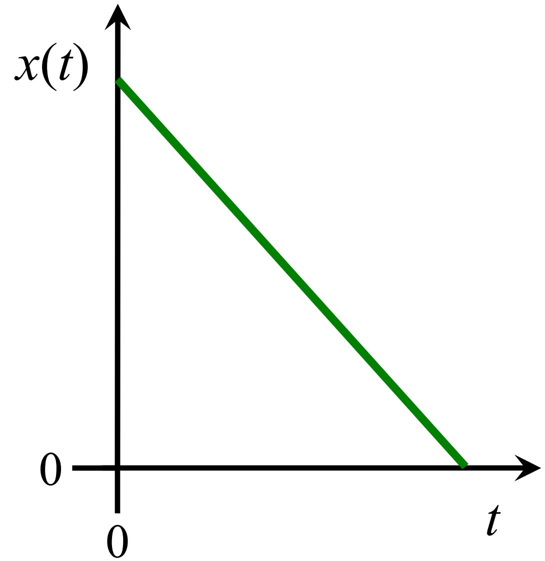

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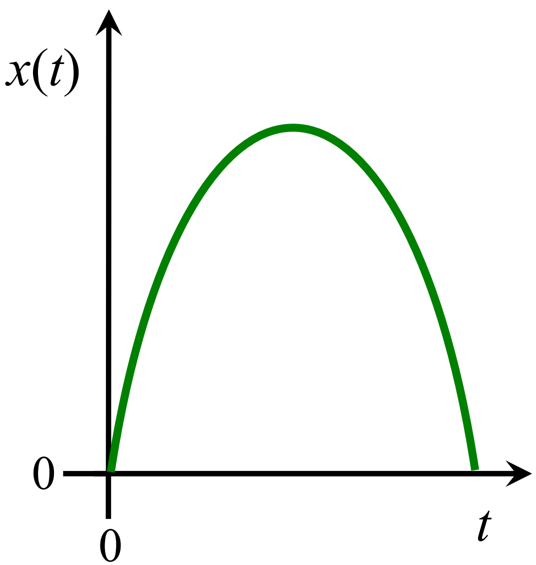
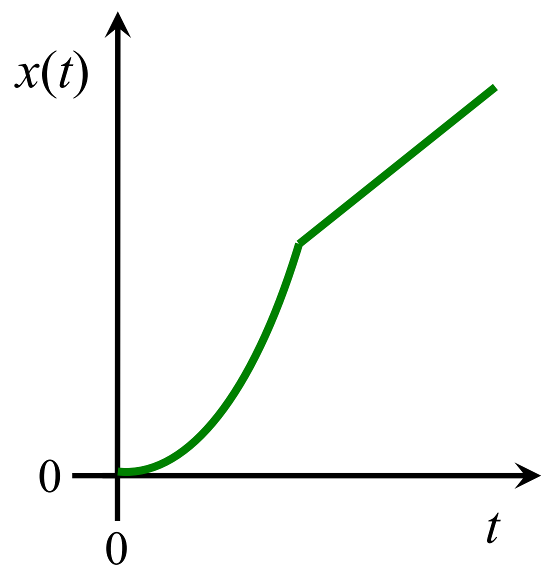


(f)



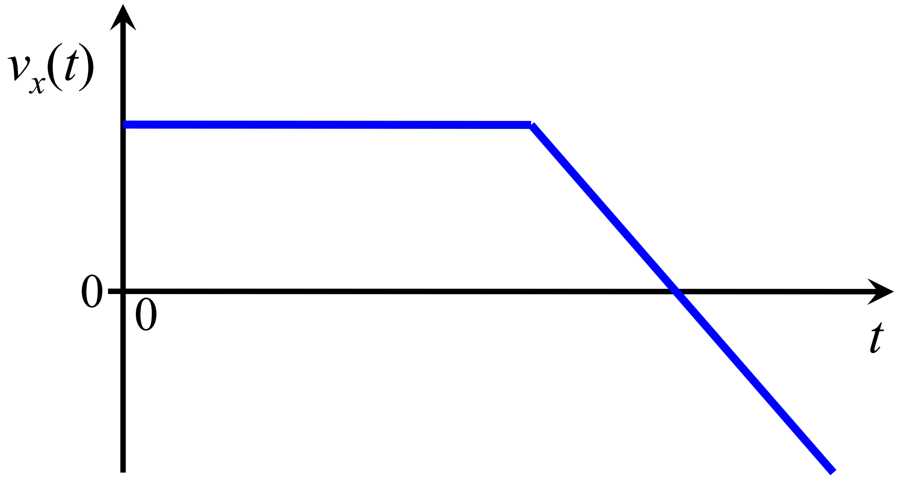
Using graphs of motion … Example 2

Consider the $v_x(t)$ versus time graph below for a car travelling along a flat, straight road.

(a) Write down everything you can say about the motion of the car.

(b) Draw the freeze frame photographic representation of the motion (use a circle for the car).

(c) Use the photographs and the velocity-time graph to sketch the corresponding **$x(t)$ vs t** and **$a_x(t)$ vs t** graphs.



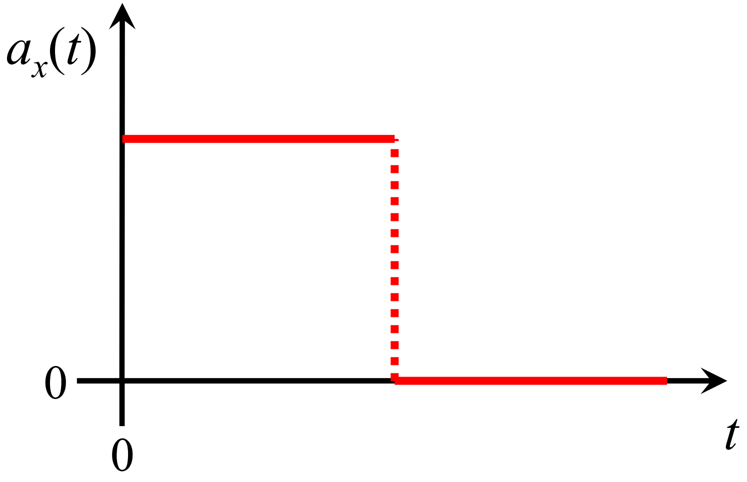
Using graphs of motion … Example 3

Consider the $a_x(t)$ versus time graph below for the motion of a train in a horizontal direction.

(a) Write down everything you can say about the motion of the train.

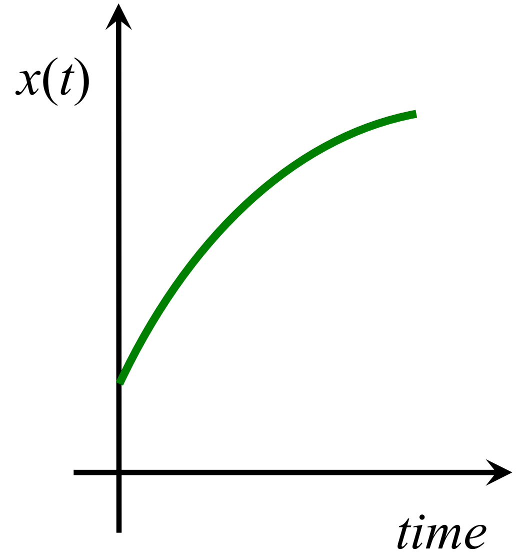
(b) Draw the freeze frame photographic representation of the motion (use a circle for the train).

(c) Use the photographs and the acceleration-time graph to sketch the corresponding **$x(t)$ vs t** and **$v_x(t)$ vs t** graphs.



FIGURING PHYSICS

A car moves along a straight road. The graph alongside shows the position of the car as a function of time.



The graph shows that the car:

(A) speeds up all the time
(B) slows down all the time
(C) moves at a constant velocity

Important worked example of motion in a straight line

A car is initially at rest. It starts to move, accelerating uniformly, and reaches a speed of 15 m s^{-1} after 20 s. It travels at a constant speed for 2 minutes after which time it slows down uniformly to stop in 30 seconds.
Assume that the entire motion takes place in a straight line in the $\hat{\mathbf{i}}$ -direction.

Determine:

(a) the average acceleration of the car during

(i) the first 20 seconds;

(ii) the last 30 seconds; and

(iii) the whole trip.

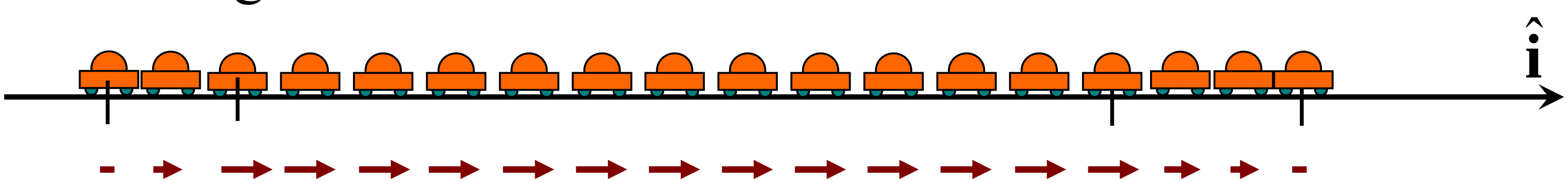
(b) the total displacement of the car.

First draw a **picture** of what is happening.
Add in a coordinate system and include all known and unknown variables ...

$a = ?$ $\quad$ $a = 0$ $\quad$ $a = ?$

$t = 0$	$t = 20$ s	$t = 140$ s	$t = 170$ s
$x(t = 0) = 0$	$x(t = 20) = ?$	$x(t = 140) = ?$	$x(t = 170) = ?$
$v_x(t = 0) = 0$	$v(t = 20) = 15\ \text{m s}^{-1}$	$v_x(t = 140) = 15\ \text{m s}^{-1}$	$v_x(t = 170) = 0\ \text{m s}^{-1}$

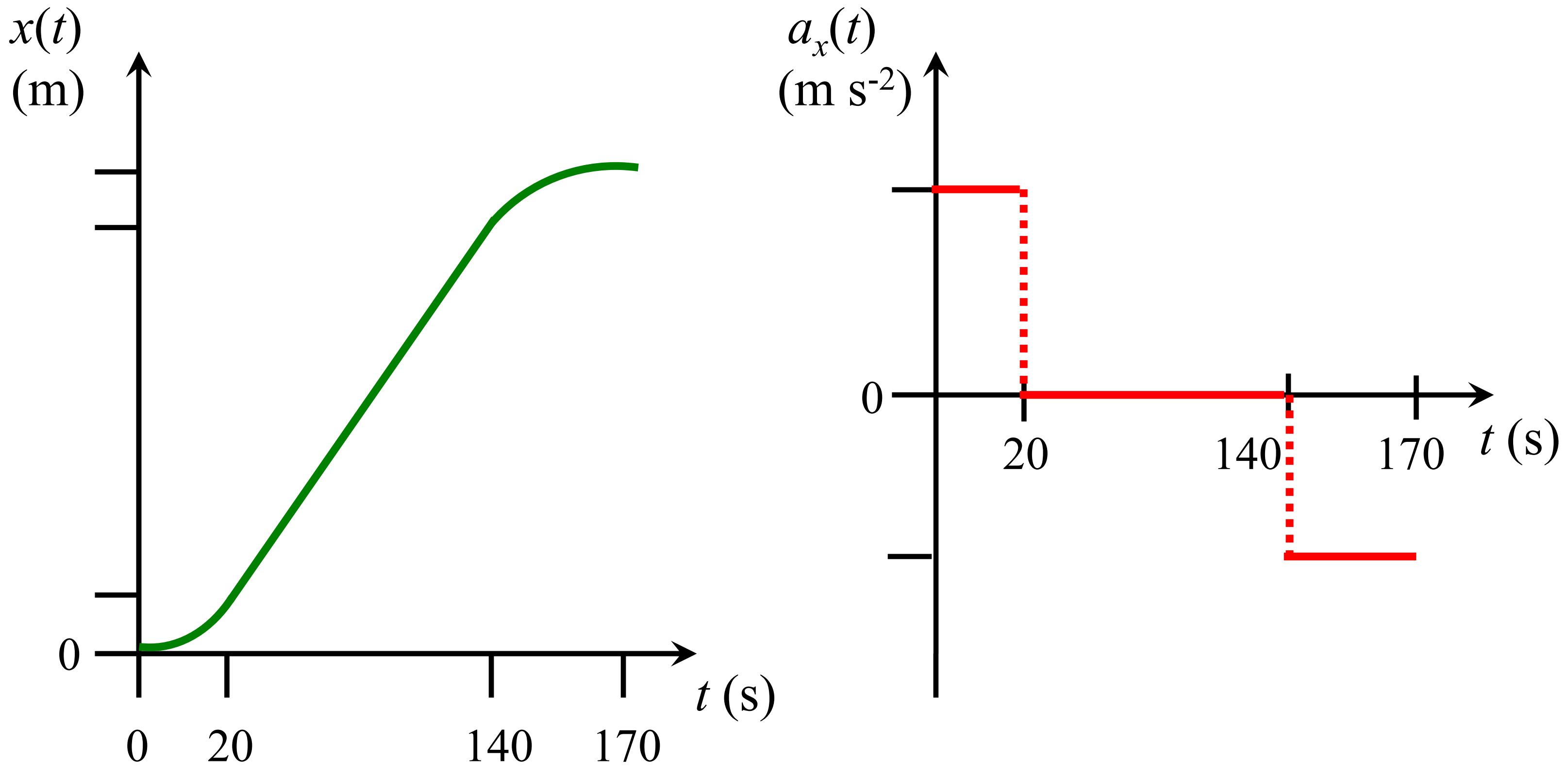
If we look a photograph of the car every 10 s then we would see something like this …



… and drawing the velocity vectors at each position.

What can you say about the direction of the **acceleration** in each region?

We can draw graphs of motion …



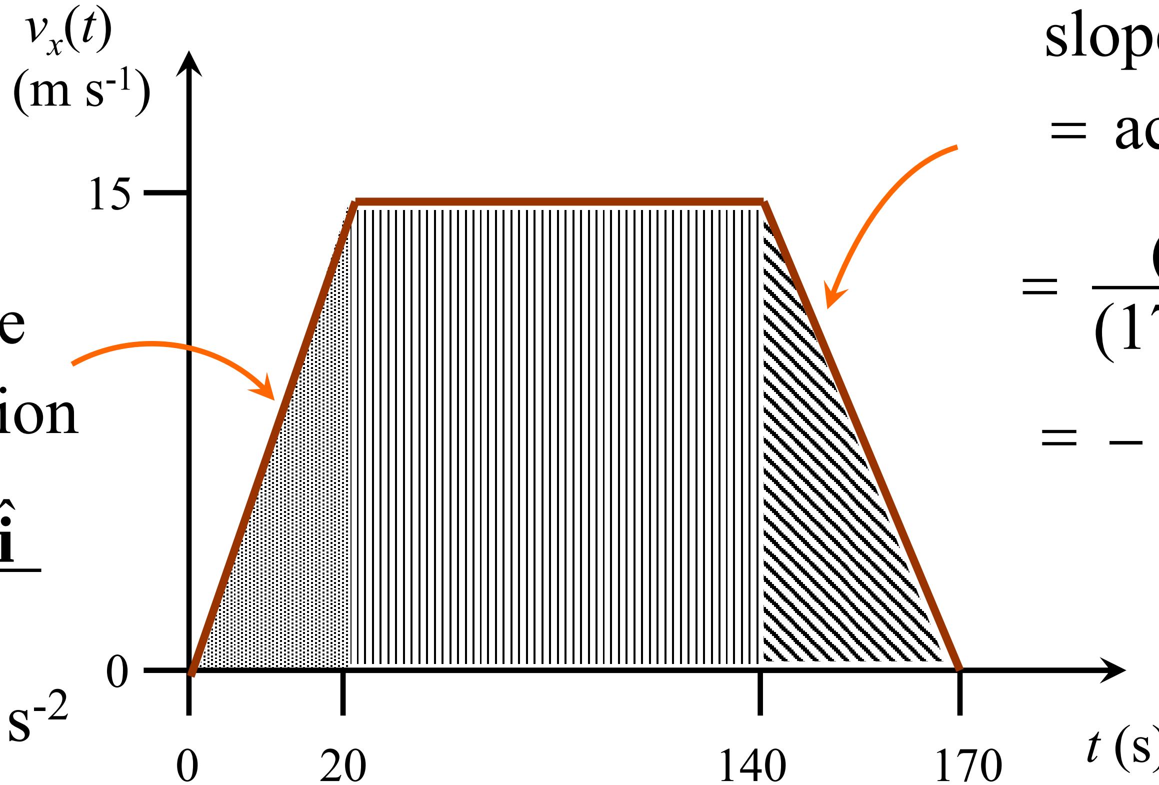
...and $v_x(t)$ versus t … next slide …

slope of line

= acceleration

$$= \frac{(15-0)\,\hat{\mathbf{i}}}{(20-0)}$$

$$= 0.75\,\hat{\mathbf{i}}\ \mathrm{m\ s^{-2}}$$

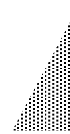
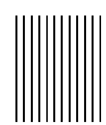
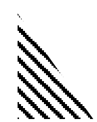


slope of line

= acceleration

$$= \frac{(0-15)\,\hat{\mathbf{i}}}{(170-140)}$$

$$= -0.50\,\hat{\mathbf{i}}\ \mathrm{m\ s^{-2}}$$

displacement = area under graph =  +  +  $= 2175\,\hat{\mathbf{i}}\ \mathrm{m}$

Finally, use the **equations of motion**:

First calculate the accelerations:

(i) Between $t = 0$ and $t = 20$ s:

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{15\hat{\mathbf{i}} - 0}{20 - 0} = 0.75\hat{\mathbf{i}} \quad \text{m s}^{-2}$$

Between $t = 20$ and $t = 140$ s: $\vec{\mathbf{a}}_{av} = 0 \ \text{m s}^{-2}$

(ii) Between $t = 140$ and $t = 170$ s:

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{0 - 15\hat{\mathbf{i}}}{170 - 140} = -0.50\hat{\mathbf{i}} \quad \text{m s}^{-2}$$

(iii) Between $t = 0$ and $t = 170$ s:

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{0 - 0}{170 - 0} = 0$$

Now the displacements ...
We consider each of the three stages separately.

(i) Between $t = 0$ and $t = 20$ s:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_0 + \vec{\mathbf{v}}_{x0}t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2$$

$$\vec{\mathbf{x}}(t=20) = (0) + (0) + \tfrac{1}{2}(0.75\hat{\mathbf{i}})(20)^2 = 150\hat{\mathbf{i}}\text{ m}$$

(ii) Between $t = 20$ and $t = 140$ s:

$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_0 + \vec{\mathbf{v}}_{x0}t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2$$

$$\vec{\mathbf{x}}(t=140) = (150\hat{\mathbf{i}}) + (15\hat{\mathbf{i}})(120) + \tfrac{1}{2}(0)(120)^2 = 1950\hat{\mathbf{i}}\text{ m}$$

(iii) Between $t = 140$ and $t = 170$ s:

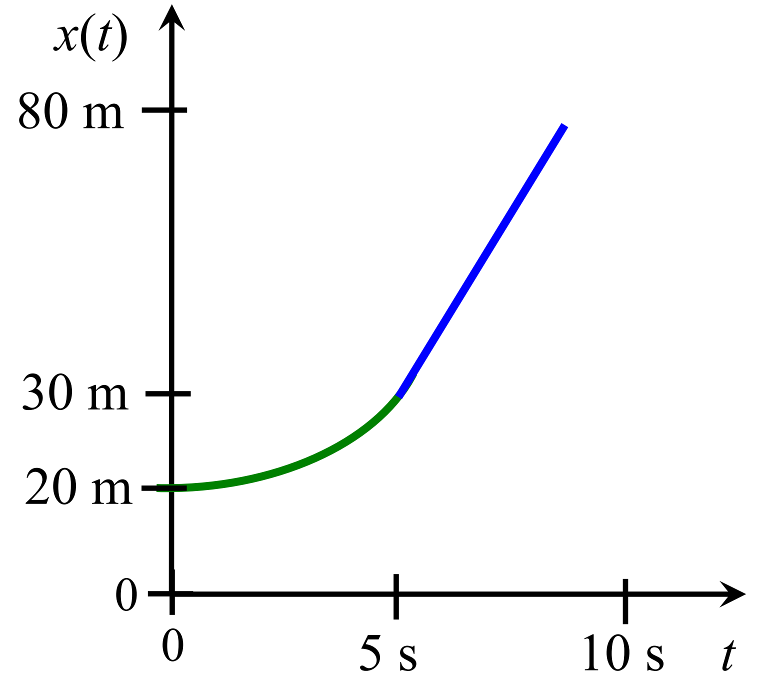
$$\vec{\mathbf{x}}(t) = \vec{\mathbf{x}}_0 + \vec{\mathbf{v}}_{x0}t + \tfrac{1}{2}\vec{\mathbf{a}}_x t^2$$

$$\vec{\mathbf{x}}(t=170) = (1950\hat{\mathbf{i}}) + (15\hat{\mathbf{i}})(30) + \tfrac{1}{2}(-0.5\hat{\mathbf{i}})(30)^2 = 2175\hat{\mathbf{i}}\text{ m}$$

Final position of the car

Using graphs of motion … Example 4

The $x(t)$-t graph for the motion of an object is shown:



(a). Write down (using words) everything you can say about the motion of the object.

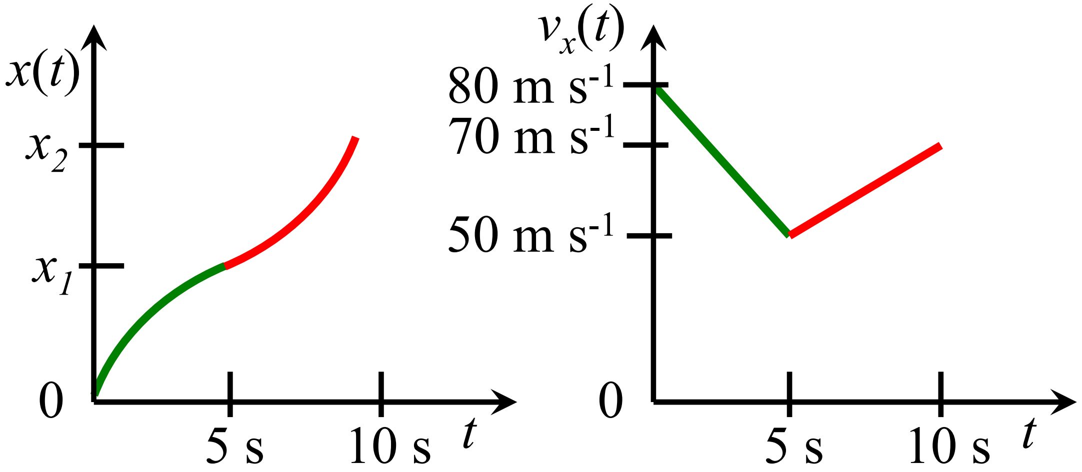
(b). Draw the freeze frame representation for each stage of the motion.

(c). Sketch the physical situation. Label your diagram clearly with all the information about the motion of the object.

(d). Determine the magnitude and direction of the acceleration of the object during the first 5 seconds.

Using graphs of motion … Example 5

The $x(t)$ vs t and $v_x(t)$ vs t graphs for the motion of an object are shown:



(a). Write down (using words) everything you can say about the motion of the object from the graphs.

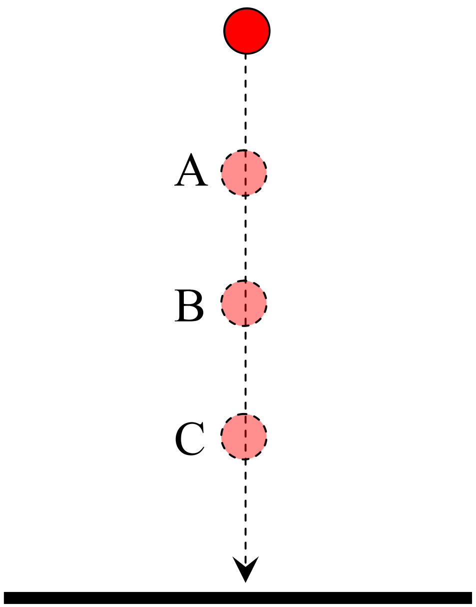
(b). Draw the freeze frame representation for each stage of the motion.

(c). Sketch the physical situation. Label your diagram clearly with all the information about the motion of the object.

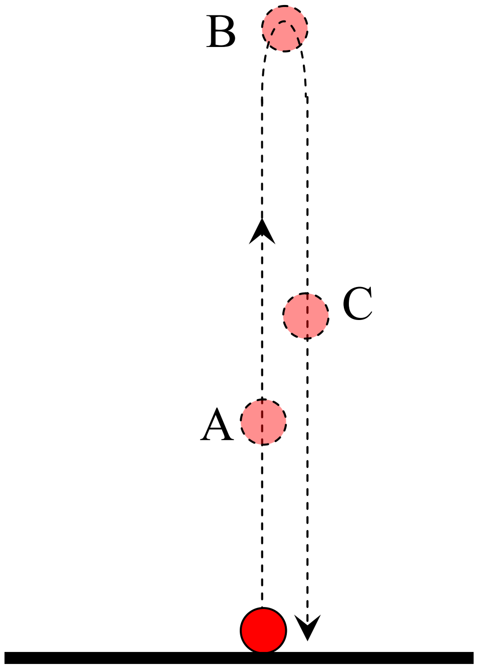
(d). Determine the total displacement of the object.

Bodies in free fall

Consider the following two situations. At each position shown, indicate the magnitude and direction of the **resultant acceleration** of the ball.



The ball is dropped from rest from a height and allowed to fall to the floor



The ball is thrown upwards, reaches some height, and falls back to the floor

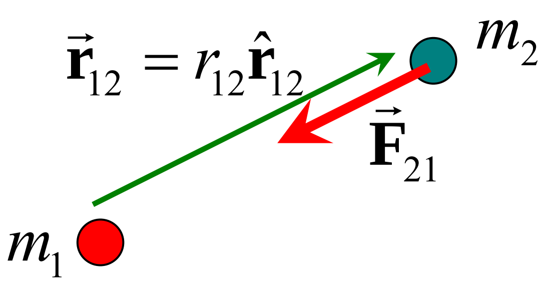
Newton's Law of Universal Gravitation.

Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. The force acts along the line joining the two particles.

$$\vec{\mathbf{F}}_{21} = G\frac{m_1 m_2}{r_{12}^2}(-\hat{\mathbf{r}}_{12})$$

force on 2 due to 1

$$\vec{\mathbf{r}}_{12} = r_{12}\hat{\mathbf{r}}_{12}$$



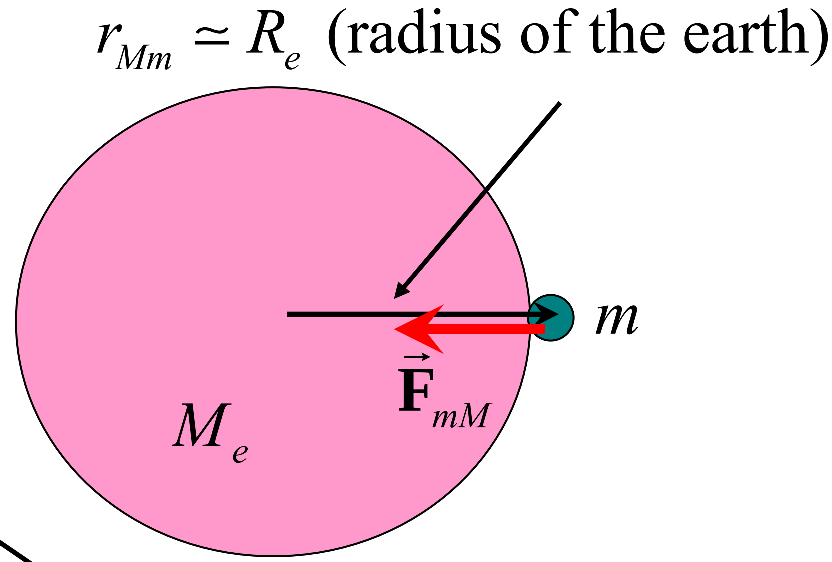
G : Universal gravitational constant.

... measured to be 6.672×10^{-11} N m^2 kg^{-2}

By Newton III: $\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$ ... see later

If one of the objects is the earth ...

$$\vec{\mathbf{F}}_{mM} = G\frac{mM_e}{R_e^2}(-\hat{\mathbf{r}}_{Mm})$$



By Newton II: $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$

$$\therefore \vec{\mathbf{F}}_{mM} = G\frac{mM_e}{R_e^2}(-\hat{\mathbf{r}}_{Mm}) = m\vec{\mathbf{a}}$$

... introduce the "local gravitation strength" $\vec{\mathbf{g}}$

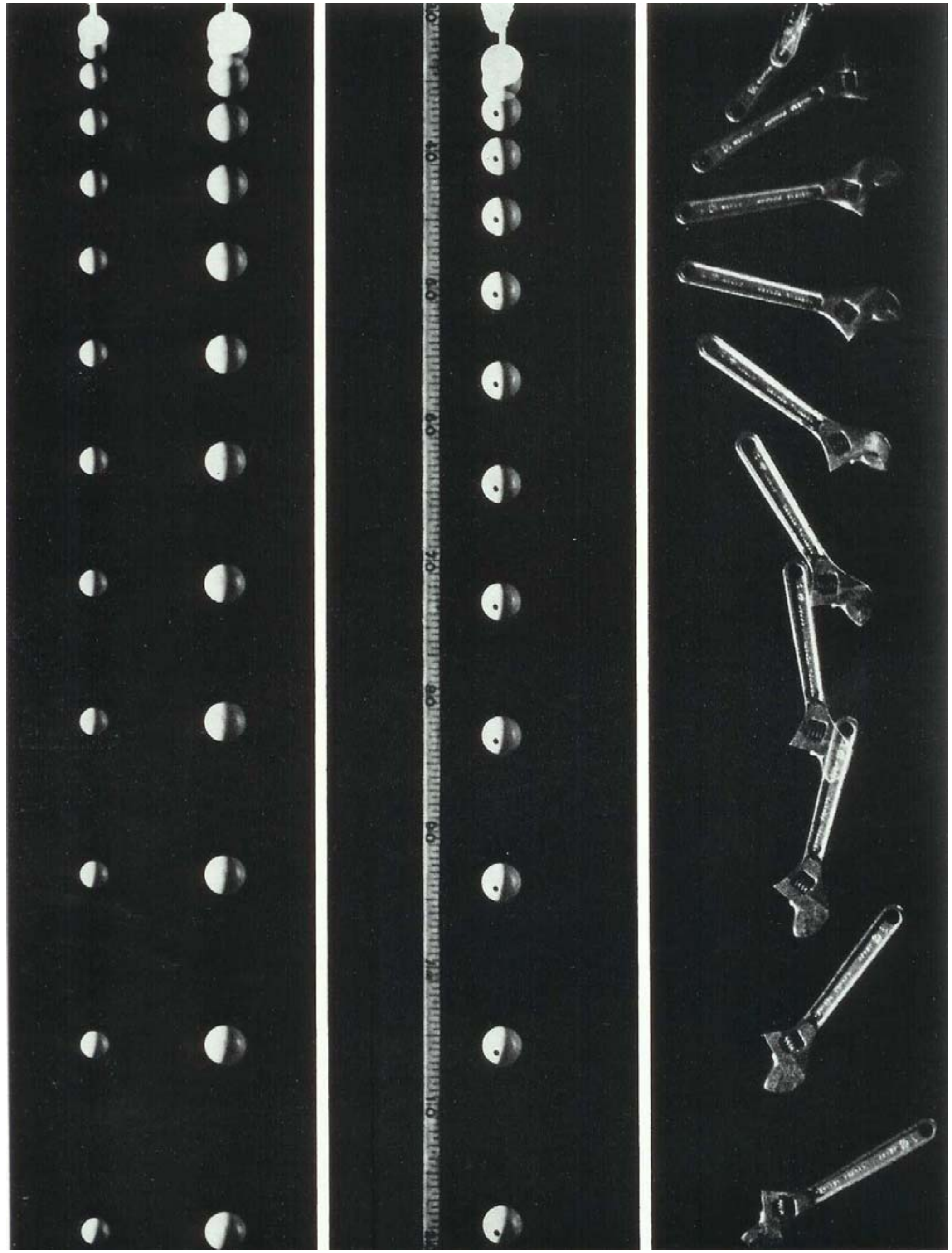
where $g = G\dfrac{M_e}{R_e^2}$ = 9.80 m s^{-2} in Cape Town

and write $\vec{\mathbf{F}}_{grav} = \vec{\mathbf{W}} = m\vec{\mathbf{g}}$

W is called the "weight" of the object.

Demonstration

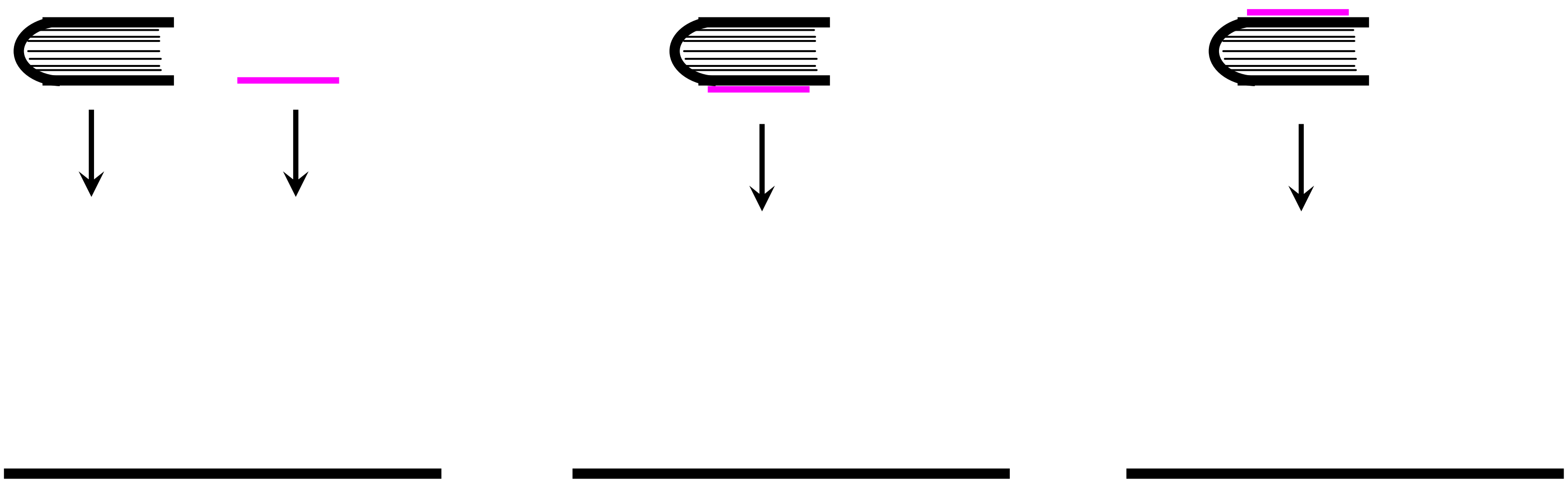
A large ball and a small ball are dropped from the same height in air. Which ball reaches the ground first?



Demonstration

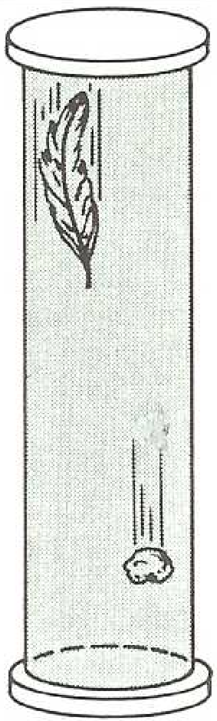
A book and a sheet of paper are dropped from the same height in air. (a) Which reaches the ground first? Does it make a difference if the paper is placed (b) under the book? ... or (c) above the book?

(a) (b) (c)

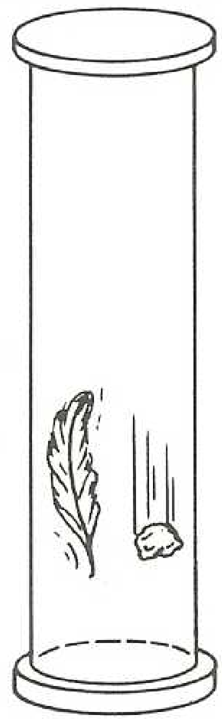


Demonstration

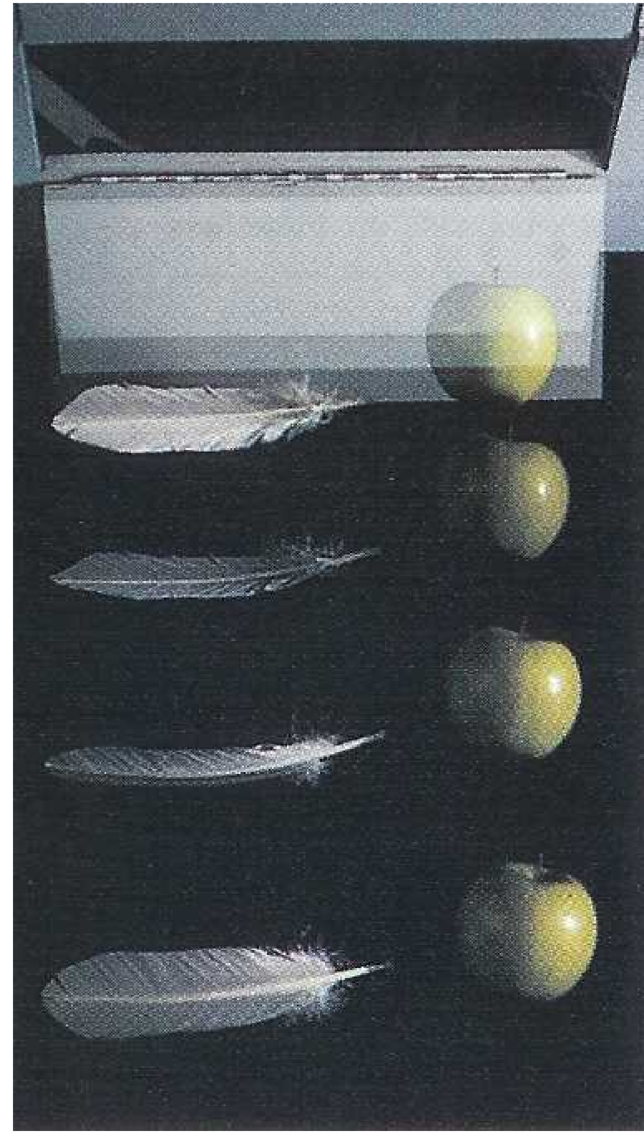
A coin (or a stone) and a disc of paper (or a feather) are dropped from a height in a cylinder filled with air. What will happen if the air is removed from the cylinder?



cylinder is filled with air



air is removed from the cylinder



FIGURING PHYSICS

If you **drop** an object in the absence of air resistance, it accelerates downward at 9.8 m s^{-2}. If instead you **throw it downward**, then its downward acceleration after release is

(A) less than 9.8 m s^{-2}
(B) 9.8 m s^{-2}
(C) more than 9.8 m s^{-2}

Look at the following in *Reese*:
Examples 3.11 page 95 ; 3.12 page 96
3.13 page 98 ; 3.14 page 99

FIGURING PHYSICS

You are throwing a ball straight up in the air.
At the highest point, the ball's ...

(A) velocity and acceleration are zero
(B) velocity is non-zero but its acceleration is zero
(C) acceleration is non-zero but its velocity is zero

Example: Throwing a ball upwards

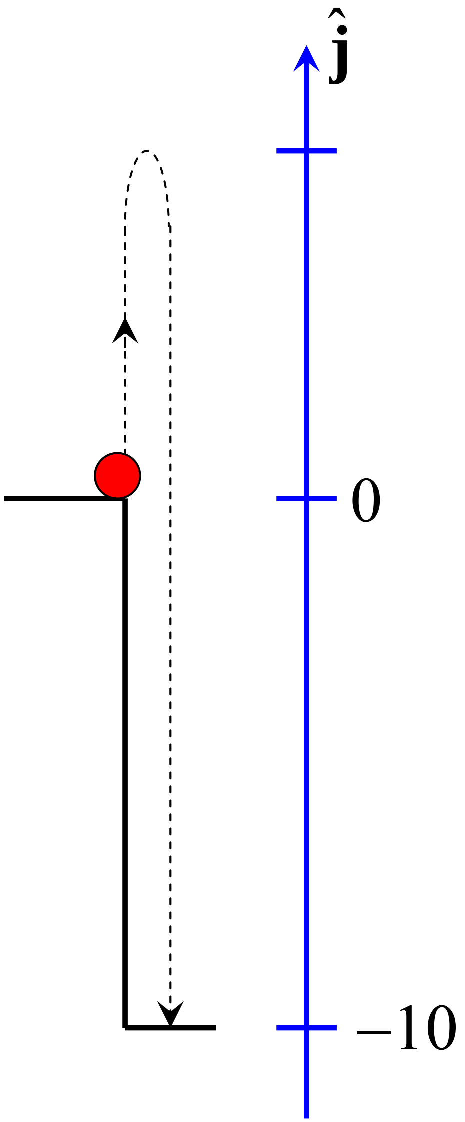
Say that you throw a ball vertically upward at 7 m s^{-1} from a balcony that is 10 m above the ground. After reaching a maximum height, the ball drops past the balcony, to the ground.

We choose a coordinate system with the $\hat{\mathbf{j}}$ direction upwards and the origin at the balcony.

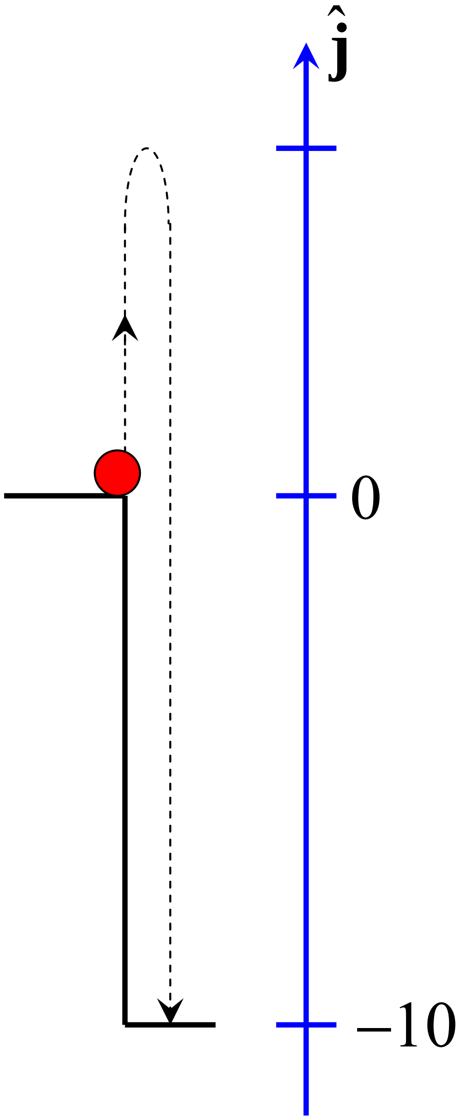
Complete the freeze frame representation of the motion of the ball …

… and also draw a **velocity vector** at each of these positions of the ball.

What is the acceleration of the ball at each of the positions shown?



Say that you throw a ball vertically upward at 7 m s^{-1} from a balcony that is 10 m above the ground. After reaching a maximum height, the ball drops past the balcony, to the ground.
With reference to the coordinate axis given …



Initial position of the ball =

Final position of the ball =

Displacement of the ball when it reaches the ground =

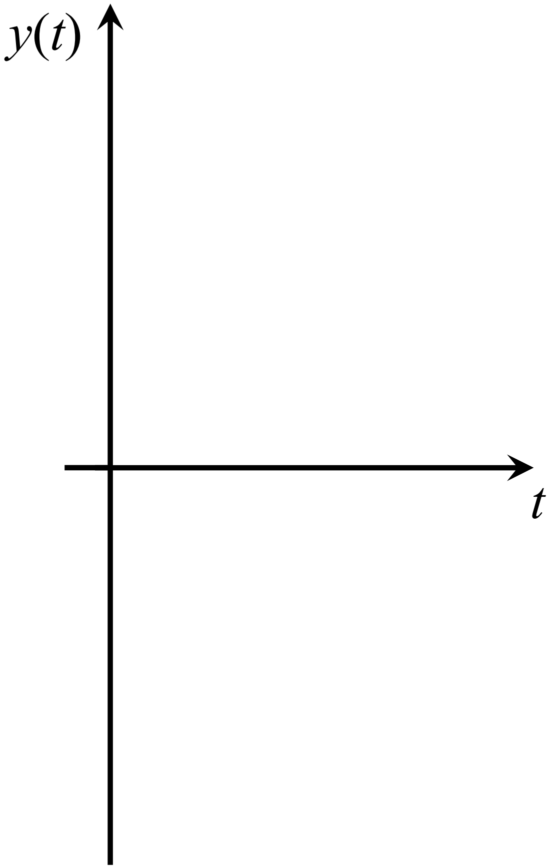
Initial velocity of the ball =

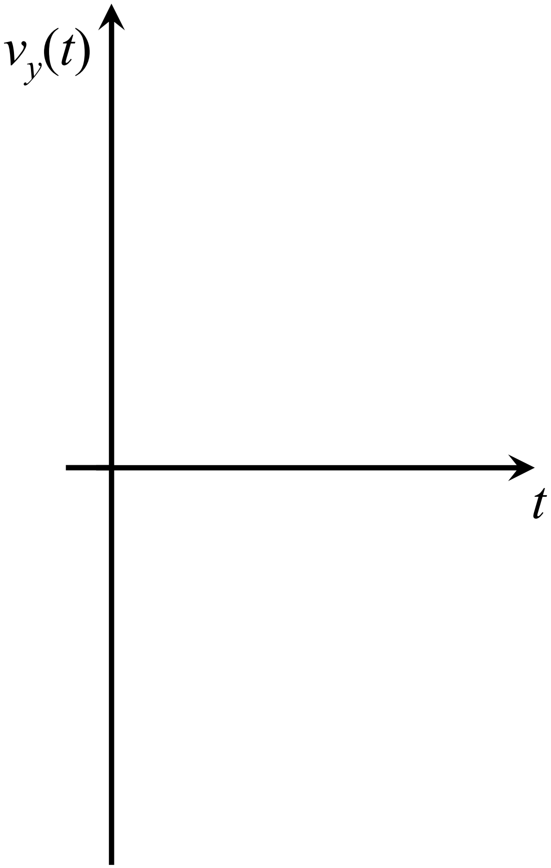
Acceleration of the ball while traveling upward =

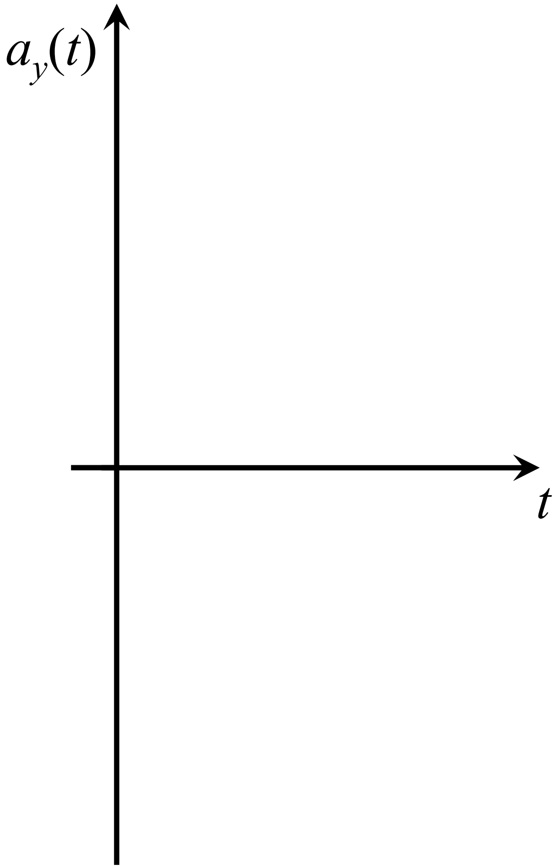
Acceleration of the ball while traveling downward =

Acceleration of the ball at maximum height =

Say that you throw a ball vertically upward at 7 m s^{-1} from a balcony that is 10 m above the ground. After reaching a maximum height, the ball drops past the balcony, to the ground.
Draw the three **graphs of motion** …







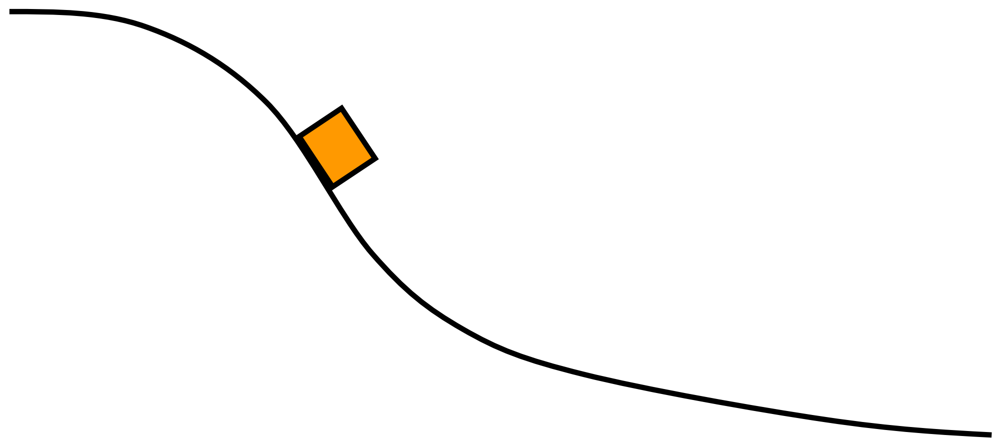
Say that you throw a ball vertically upward at 7 m s^{-1} from a balcony that is 10 m above the ground. After reaching a maximum height, the ball drops past the balcony, to the ground.
Now determine how long it takes for the ball to reach the ground.

FIGURING PHYSICS

A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed.
Neglecting air resistance, the ball to hit the ground below the cliff with the greater speed is the one initially thrown

(A) upward
(B) downward
(C) neither - they both hit at the same speed.

Figuring Physics



A box slides on a frictionless surface as shown above.
As the box moves beyond the point shown, what happens to its speed and acceleration in the direction of motion?

(A) both decrease
(B) the speed increases, but acceleration decreases
(C) the speed decreases, but acceleration increases

Problems involving two moving bodies

The general approach when dealing with two moving bodies is to apply a **single set of coordinate axes** to a diagram of the situation, draw freeze frame representations to understand the situation qualitatively, and then only apply the equations of motion separately to each object.
The equations will be linked to each other usually by one or more parameters, such as time, or the final positions (if they are the same for each object).

Example 1

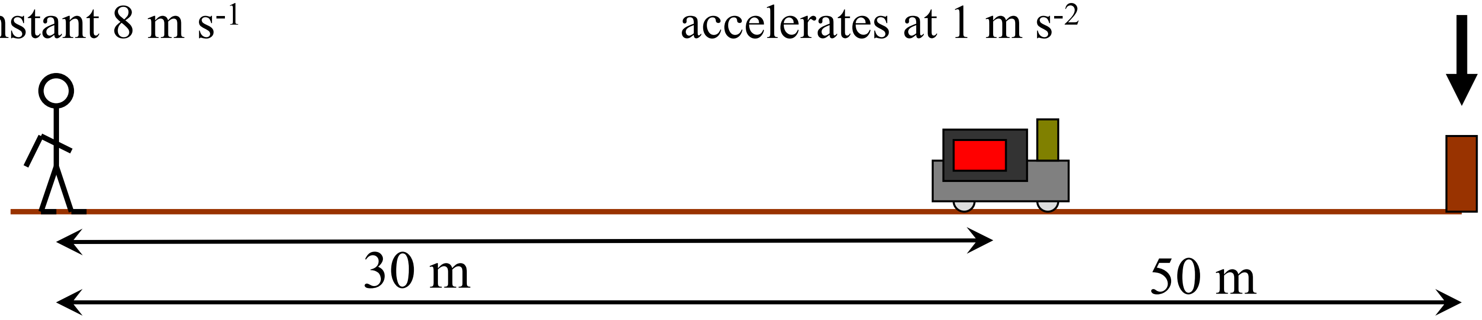
You want to visit your friend in Durban over the Winter vacation. To save money, you decide to travel there by train. But you are late finishing your physics exam, so you are late in arriving at the train station. You run as fast as you can, but just as you reach one end of the platform your train departs, 30 metres ahead of you down the platform. You can run at a maximum speed of 8 m s^{-1} and the train is accelerating at 1 m s^{-2}. You can run along the platform for 50 m before you reach a barrier. Will you catch your train?

Pictorial representation

Student runs at a constant 8 m s^{-1}

train starts from rest and accelerates at 1 m s^{-2}

end of platform (will they meet before here?)



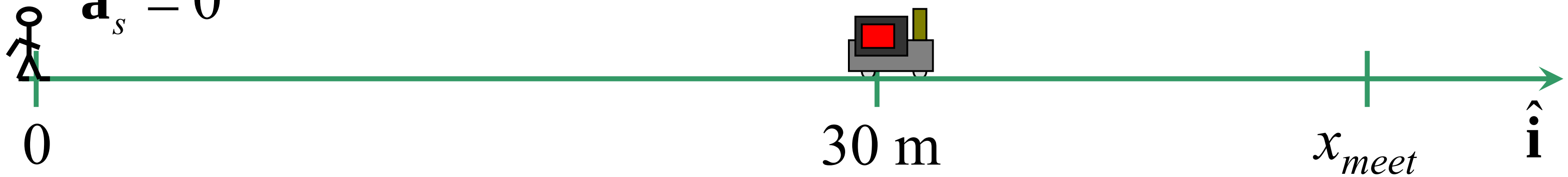
Physics representation

$\vec{\mathbf{v}}_{0s} = 8\,\hat{\mathbf{i}}\ \text{ m s}^{-1}$

$\vec{\mathbf{a}}_s = 0$

$\vec{\mathbf{v}}_{0t} = 0$

$\vec{\mathbf{a}}_t = 1\,\hat{\mathbf{i}}\ \text{ m s}^{-2}$



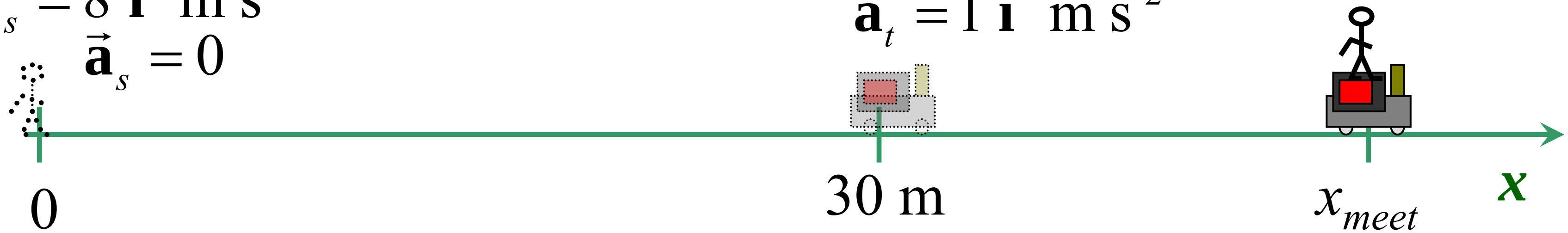
Physics representation

$\vec{\mathbf{v}}_{0s} = 8\,\hat{\mathbf{i}}\ \ \text{m s}^{-1}$

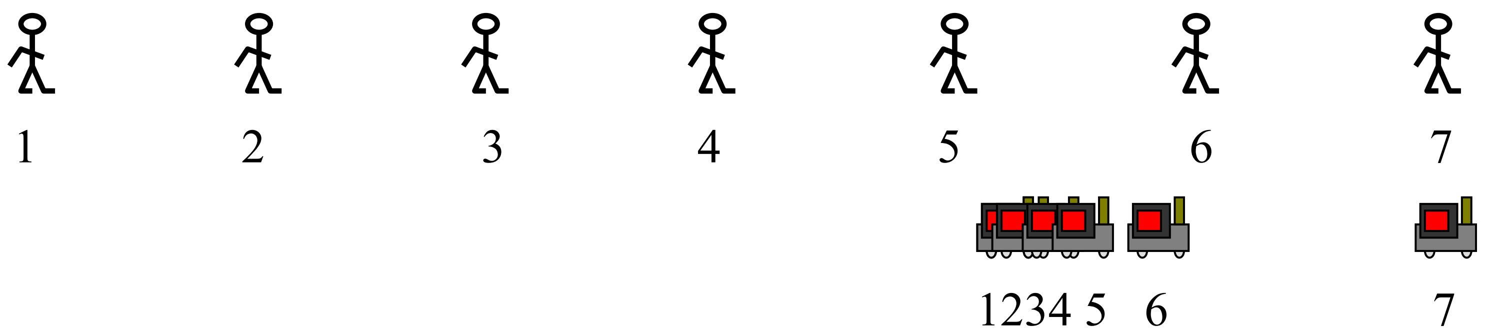
$\vec{\mathbf{a}}_s = 0$

$\vec{\mathbf{v}}_{0t} = 0$

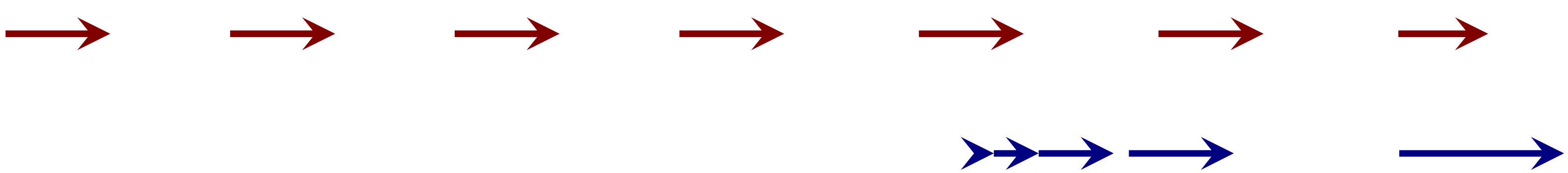
$\vec{\mathbf{a}}_t = 1\,\hat{\mathbf{i}}\ \ \text{m s}^{-2}$



Freeze frame:



Velocity vectors:



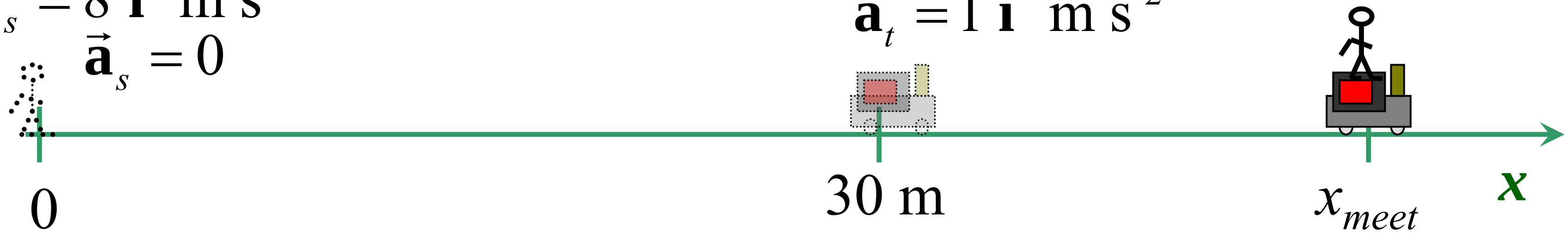
Physics representation

$\vec{\mathbf{v}}_{0s} = 8\ \hat{\mathbf{i}}\ \ \text{m s}^{-1}$

$\vec{\mathbf{a}}_s = 0$

$\vec{\mathbf{v}}_{0t} = 0$

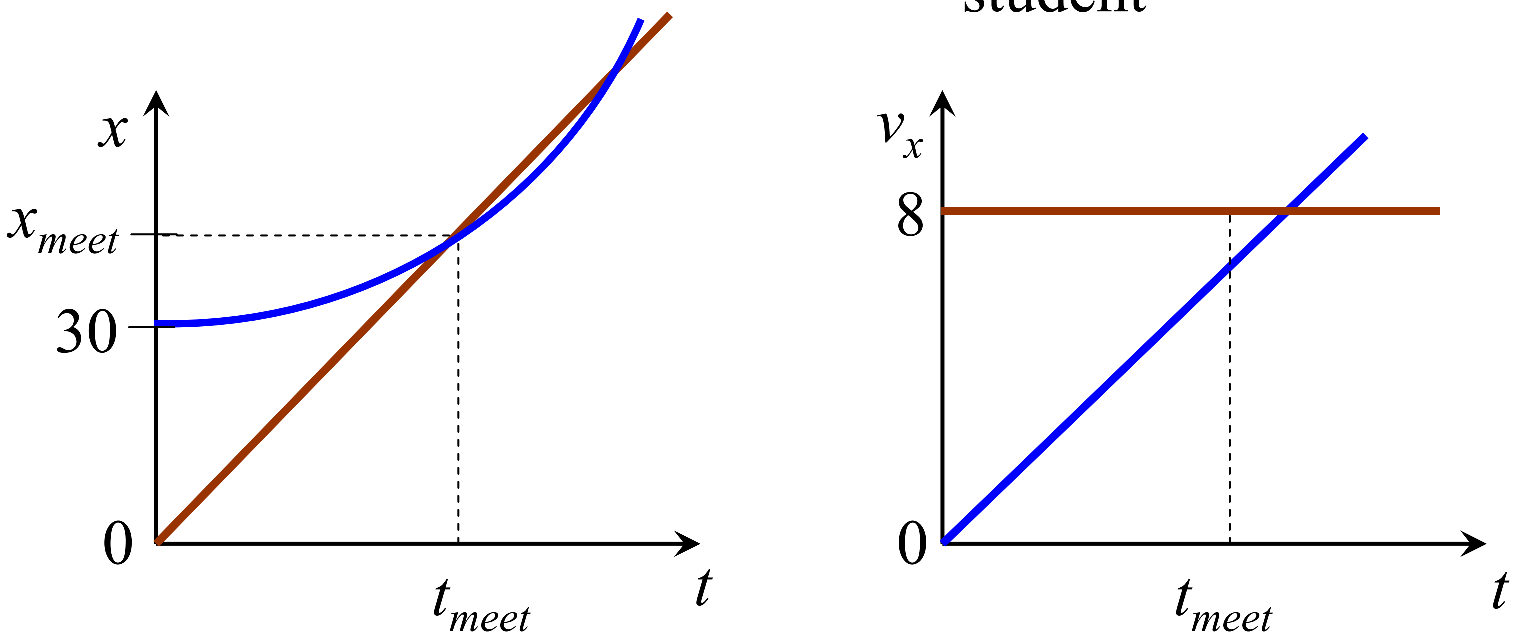
$\vec{\mathbf{a}}_t = 1\ \hat{\mathbf{i}}\ \ \text{m s}^{-2}$



Graphs of motion:

train

student



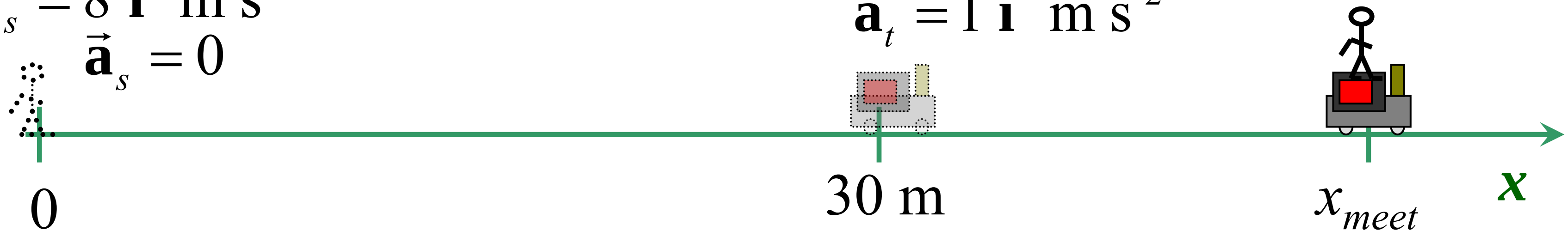
Physics representation

$\vec{\mathbf{v}}_{0s} = 8\,\hat{\mathbf{i}}\ \ \mathrm{m\ s^{-1}}$

$\vec{\mathbf{a}}_s = 0$

$\vec{\mathbf{v}}_{0t} = 0$

$\vec{\mathbf{a}}_t = 1\,\hat{\mathbf{i}}\ \ \mathrm{m\ s^{-2}}$



Mathematics representation

<u>Student</u>

$$\vec{\mathbf{x}}_s(t) = \vec{\mathbf{x}}_{0s} + \vec{\mathbf{v}}_{0s}t + \tfrac{1}{2}\vec{\mathbf{a}}_s t^2$$

$$x_{meet}\hat{\mathbf{i}} = 0\hat{\mathbf{i}} + 8\hat{\mathbf{i}}t + 0\hat{\mathbf{i}}$$

$$x_{meet} = 0 + 8t + 0$$

$$x_{meet} = 8t$$

<u>Train</u>

$$\vec{\mathbf{x}}_t(t) = \vec{\mathbf{x}}_{0t} + \vec{\mathbf{v}}_{0t}t + \tfrac{1}{2}\vec{\mathbf{a}}_t t^2$$

$$x_{meet}\hat{\mathbf{i}} = 30\hat{\mathbf{i}} + 0\hat{\mathbf{i}} + \tfrac{1}{2}(1\hat{\mathbf{i}})t^2$$

$$x_{meet} = 30 + 0 + \tfrac{1}{2}(1)t^2$$

$$x_{meet} = 30 + \tfrac{1}{2}t^2$$

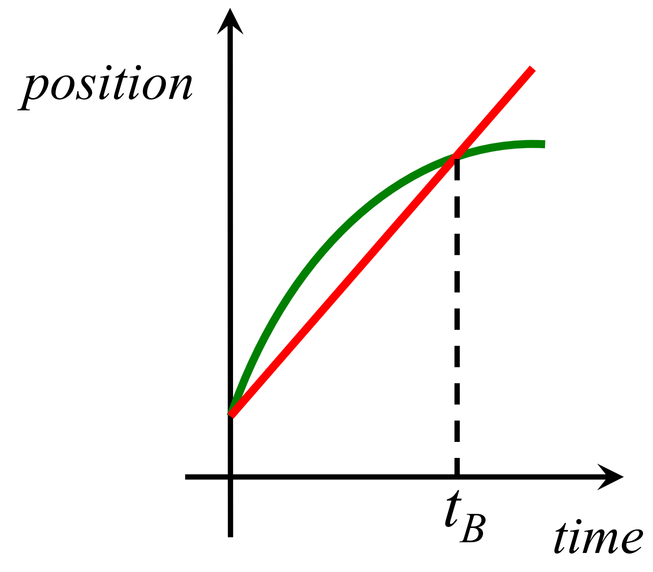
… Solve for t and x_{meet}

… find $x_{meet} = 48$ m which is less than 50 m

… so you will catch the train before the end of the platform!

Figuring Physics

The graph shows position as a function of time for two trains running on parallel tracks. Which is true:



(A) At time t_B both trains have the same velocity
(B) Both trains speed up all the time
(C) Both trains have the same velocity at some time before t_B

Example 2

You are driving at a speed of 60 km h^{-1} and see a truck 20 m ahead coming directly towards you at a constant speed of 40 km h^{-1}. If you immediately hit the breaks and your car starts to slow down at 8.0 m s^{-2}, how long is it before the truck smashes into you?

Example 2 … continued

Choose the origin at the car with the $\hat{\mathbf{i}}$ unit vector pointing to the right.

(a) What can you say about the motion of the car and the truck? Use words!
(b) Sketch the physical situation. Draw the car and the truck at their initial and final positions.
(c) Draw a coordinate axis on your diagram. Label the initial and final positions of the car and the truck.
(d) What is the initial velocity of the car?
(e) What is the initial velocity of the truck?
(f) Sketch the freeze frame representation for the car and the truck.
(g) On your "freeze frame" representation draw in the paths of the car and the truck.
(h) Draw in vectors at each of the seven positions to indicate the direction and magnitude for the velocity of the car and truck.
(i) What is the acceleration of the car?
(j) What is the acceleration of the truck?

Example 2 … continued

(k) Sketch, on the same set of axes, **velocity-time** graphs for the car and the truck. Indicate clearly the point where the truck and the car will meet.
(l) Sketch, on the same set of axes, **position-time** graphs for the car and the truck. Indicate clearly the point where the truck and the car will meet.
(m) Use your graphs to determine the time before the truck smashes into the car.
(n) Now use the **equations of motion** to calculate this time (and the position of the collision).
(o) Write down the conditions (in terms of velocity and acceleration) under which there will be no collision between the car and the truck, i.e. the car and the truck will just touch for an instant before moving apart again. Explain this in terms of your graphs of motion.

Example 3

You and your brother are playing a game on the beach. He is standing on a ledge 15 m above you (standing on the beach) and throws a ball vertically upward with an initial speed of 30 m s^{-1}. The idea is for you to simultaneously throw a stone vertically upward from the beach so that it hits the ball at the apex of its flight. At what speed should you throw the stone?

Example 3 … continued

Choose the origin at the beach with the $\hat{\mathbf{j}}$ unit vector pointing upwards.

(a) What can you say about the motion of the stone and the ball? Use words!
(b) Sketch the physical situation. Draw the stone and the ball at their initial and final positions.
(c) Draw a coordinate axis on your diagram. Label the initial and final positions of the stone and the ball.
(d) What is the initial velocity of the stone?
(e) What is the initial velocity of the ball?
(f) Sketch the freeze frame representation for the stone and the ball.
(g) On your "freeze frame" representation draw in the paths of the stone and the ball.
(h) Draw in vectors at each of the seven positions to indicate the direction and magnitude for the velocity of the stone and ball.

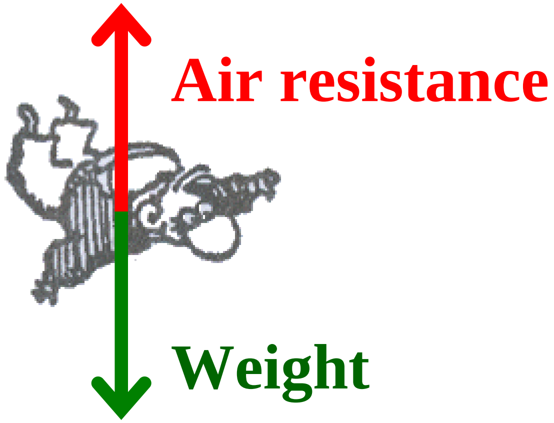
Example 3 … continued

(i) What is the acceleration of the stone?
(j) What is the acceleration of the ball?
(k) Sketch, on the same set of axes, **velocity-time** graphs for the stone and the ball. Indicate clearly the point where the stone and the ball will meet.
(l) Sketch, on the same set of axes, **position-time** graphs for the stone and the ball. Indicate clearly the point where the stone and the ball will meet.
(m) Use your graphs to determine the initial velocity of the stone.
(n) Now use the **equations of motion** to calculate this initial velocity (and the position of the collision).

A note on air resistance and terminal velocity

When an object begins to fall through a fluid (such as a skydiver falling through the air), initially the only force acting on the object is gravity, and its acceleration is *g*.
As the object picks up speed, the frictional force ("air resistance" in air) increases, and the acceleration of the object decreases in magnitude.

Eventually the two forces will be balanced, and the object falls at a constant velocity, called the **terminal velocity**.



For a skydiver, terminal velocity is between 160 to 240 km h^{-1}.

An approach to solving physics problems

Step 1. Think carefully about the problem situation and draw a picture of what is going on (Pictorial Representation).

- Draw one or more **pictures** which show all the important objects, their motion and any interactions.
- Now ask "**What is being asked**?" "Do I need to calculate something?"
- Think about what **physics concepts** and principles you think will be useful in solving the problem and when they will be most useful.
- Construct a **mental image** of the problem situation - do your friends have the same image?
- Specify a convenient **system** to use - circle this on your picture.
- Identify any **idealisations and constraints** present in the situation - write them down!
- Specify any **approximations** or simplifications which you think will make the problem solution easier, but will not affect the result significantly.

Step 2. Describe the physics (Physics Representation).

- Draw a **coordinate axis** (or axes) onto your picture (decide where to put the origin and on the direction of the axes).
- Translate your pictures into one or more **diagrams** (with axes) which only gives the essential information for a mathematical solution.
- If you are using kinematic concepts, draw a freeze frame diagram with velocity vectors.
- If interactions or statics are important, draw idealised, free body and force diagrams.
- When using conservation principles, draw "initial" and "final" diagrams to show how the system changes.
- For optics problems draw a ray diagram.
- For circuit problems, a circuit diagram will be useful.
- Define a **symbol** for every important physics variable in your diagram and write down what information you know (e.g. $T_1 = 30$ N).
- Identify your **target** variable? ("What unknown must I calculate?").

Step 3. Represent the problem mathematically and plan a solution (Mathematical Representation).

- Only now think about what **mathematical expressions** relate the physics variables from your diagrams.
- Using these mathematical expressions, construct specific **algebraic equations** which describe the specific situation above.
- Think about **how** these equations can be combined to find your target variable.
- Begin with an equation that contains the target variable.
- Identify any unknowns in that equation
- Find equations which contain these unknowns
- Do not solve equations numerically at this time.
- Check your equations for **sufficiency**... You have a solution if your plan has as many independent equations are there are unknowns. If not, determine other equations or check the plan to see if it is likely that a variable will cancel from your equations.
- **Plan** the best order in which to solve the equations for the desired variable.

Step 4. Execute the plan

- **Do the algebra** in the order given by your outline.
- When you are done you should have a single equation with your target variable.
- **Substitute** the values (numbers with units) into this final equation.
- Make sure **units** are **consistent** so that they will cancel properly.
- **Calculate** the numerical **result** for the target variable.

Step 5. Evaluate your solution

- Do **vector** quantities have both magnitude and direction ?
- Does the **sign** of your answer make sense ?
- Can someone else follow your solution ? Is it **clear** ?
- Is the **result reasonable** and within your experience ?
- Do the **units** make sense ?

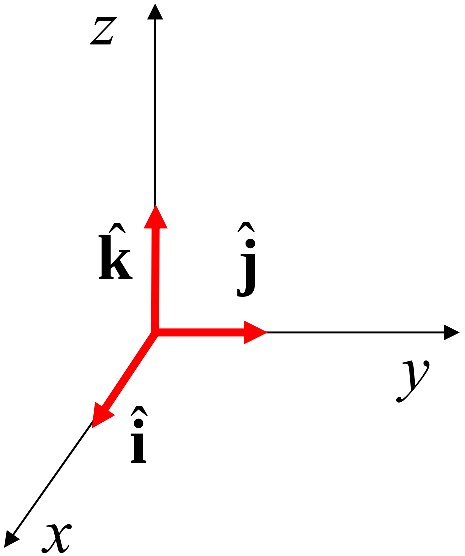
Have you answered the question ?

Summary of vector algebra

Right handed coordinate system:

Unit vectors $\hat{\mathbf{i}}$ $\hat{\mathbf{j}}$ $\hat{\mathbf{k}}$

$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$



$$\vec{\mathbf{A}} = (A_x, A_y, A_z) = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = (B_x, B_y, B_z) = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x, A_y + B_y, A_z + B_z)$$

$$-\vec{\mathbf{A}} = (-A_x, -A_y, -A_z)$$

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}}) = (A_x - B_x, A_y - B_y, A_z - B_z)$$

$$c\vec{\mathbf{A}} = (cA_x, cA_y, cA_z)$$

$$\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z = \mathrm{d}$$

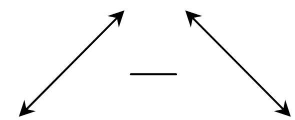
$$\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = \vec{\mathbf{B}} \bullet \vec{\mathbf{A}} \qquad \vec{\mathbf{A}} \bullet \vec{\mathbf{A}} = \mathrm{A}^2$$

$$\hat{\mathbf{i}} \bullet \hat{\mathbf{i}} = \hat{\mathbf{j}} \bullet \hat{\mathbf{j}} = \hat{\mathbf{k}} \bullet \hat{\mathbf{k}} = 1 \qquad \hat{\mathbf{i}} \bullet \hat{\mathbf{j}} = \hat{\mathbf{j}} \bullet \hat{\mathbf{k}} = \hat{\mathbf{k}} \bullet \hat{\mathbf{i}} = 0$$

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = (A_y B_z - A_z B_y)\hat{\mathbf{i}} + (A_z B_x - A_x B_z)\hat{\mathbf{j}} + (A_x B_y - A_y B_x)\hat{\mathbf{k}}$$

$$= \vec{\mathbf{G}} \quad \text{where} \quad \vec{\mathbf{G}} \perp \vec{\mathbf{A}} \quad \text{and} \quad \vec{\mathbf{G}} \perp \vec{\mathbf{B}}$$

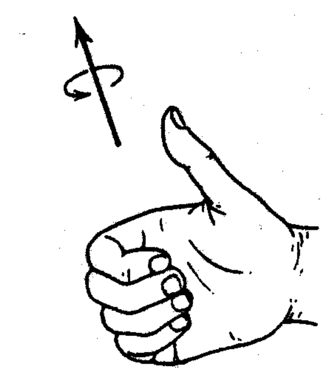
easy to remember: $\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$ always



$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = -(\vec{\mathbf{B}}\times\vec{\mathbf{A}}) \qquad \vec{\mathbf{A}}\times\vec{\mathbf{A}} = 0$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0 \qquad \hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}};\ \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}};\ \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$$

In polar form in 2D:



$$\vec{\mathbf{A}}\bullet\vec{\mathbf{B}} = AB\cos\theta \quad \text{and} \quad \vec{\mathbf{A}}\times\vec{\mathbf{B}} = AB\sin\theta\,\hat{\mathbf{k}}$$

where θ is the angle between tails of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

Differentiation of vector functions

If $\vec{\mathbf{A}}(t) = A_x(t)\hat{\mathbf{i}} + A_y(t)\hat{\mathbf{j}} + A_z(t)\hat{\mathbf{k}}$

then $$\frac{d}{dt}\vec{\mathbf{A}}(t) = \frac{dA_x(t)}{dt}\hat{\mathbf{i}} + \frac{dA_y(t)}{dt}\hat{\mathbf{j}} + \frac{dA_z(t)}{dt}\hat{\mathbf{k}}$$

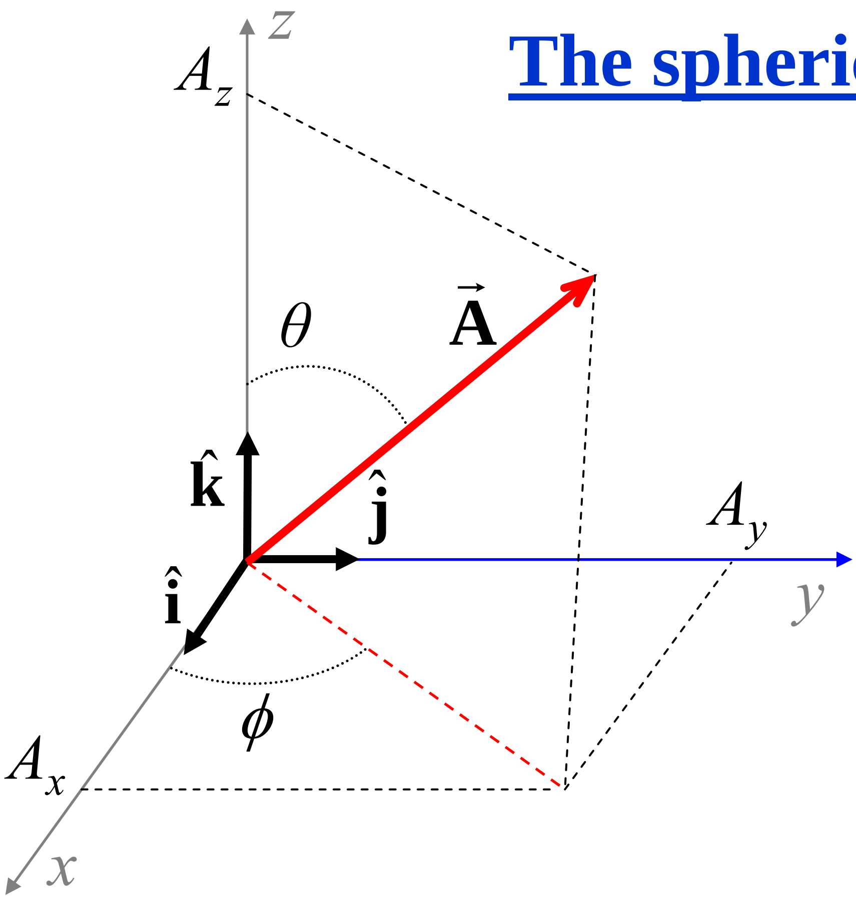
Also: $$\frac{d}{dt}\left[\vec{\mathbf{A}}(t) + \vec{\mathbf{B}}(t)\right] = \frac{d\vec{\mathbf{A}}(t)}{dt} + \frac{d\vec{\mathbf{B}}(t)}{dt}$$

$$\frac{d}{dt}\left[c(t)\vec{\mathbf{A}}(t)\right] = \frac{dc(t)}{dt}\vec{\mathbf{A}}(t) + c(t)\frac{d\vec{\mathbf{A}}(t)}{dt}$$

$$\frac{d}{dt}\left[\vec{\mathbf{A}}(t)\bullet\vec{\mathbf{B}}(t)\right] = \vec{\mathbf{A}}(t)\bullet\frac{d\vec{\mathbf{B}}(t)}{dt} + \frac{d\vec{\mathbf{A}}(t)}{dt}\bullet\vec{\mathbf{B}}(t)$$

$$\frac{d}{dt}\left[\vec{\mathbf{A}}(t)\times\vec{\mathbf{B}}(t)\right] = \vec{\mathbf{A}}(t)\times\frac{d\vec{\mathbf{B}}(t)}{dt} + \frac{d\vec{\mathbf{A}}(t)}{dt}\times\vec{\mathbf{B}}(t)$$

The spherical polar coordinate system



$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$A_x = A\ \cos\phi\ \sin\theta$$

$$A_y = A\ \sin\phi\ \sin\theta$$

$$A_z = A\ \cos\theta$$

Spherical coordinates: A, θ, ϕ :

$$\begin{cases} A = \sqrt{A_x^{\,2} + A_y^{\,2} + A_z^{\,2}} \\ \cos\theta = \dfrac{A_z}{A} \\ \tan\phi = \dfrac{A_y}{A_x} \end{cases}$$